Investment and capital structure of partially private regulated firms*

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July 25, 2011

Abstract

We develop a model that examines the capital structure and investment decisions of regulated firms in a setting that incorporates two key institutional features of the public utilities sector in many countries: firms are partially owned by the state and regulators are not necessarily independent. Among other things, we show that firms invest more, issue more debt, and are allowed to charge higher prices when they are more privatized and when the regulator is more independent and more pro-firm.

JEL Classification: G32, L33, L51,

Keywords: regulation, debt, investment, government ownership, regulatory independence, regulatory climate

*We thank seminar participants at the 2009 EARIE Conference in Ljubljana, the 2010 “International Conference on Infrastructure Economics and Development,” in Toulouse, the 2011 “Industrial Organization: Theory, Empirics and Experiments” workshop in Otranto, the 2011 International Conference on Competition and Regulation in Rhodes, FEEM in Milano, and IAE in Paris. Yossi Spiegel thanks the Henry Crown Institute of Business Research in Israel for financial assistance and Carlo Cambini gratefully acknowledges financial support from the Italian Ministry of Education (No. 20089PYFHY_004).

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1 Introduction

Since the early 1990’s, many countries around the world have substantially reformed their public utilities sector through large scale privatization and by establishing Independent Regulatory Agencies (IRAs) to regulate the newly privatized utilities. These reforms were intended to improve the efficiency and service quality of utilities and boost their investments. The structural reforms, however, were accompanied by a substantial increase in the financial leverage of regulated utilities. For example, Telefonica de Espana, the Spanish incumbent telecom operator, increased its leverage after being privatized in 1997 from 36% to 68% in 2005; Autostrade per l’Italia, the largest freight road operator in Italy, increased its leverage from 32% in 1999, when it was completely privatized, to 88% in 2003; National Grid Group Plc, the UK energy transport operator, increased its leverage from 30% in 1997 to 72% in 2005; and Anglian Water Plc, the largest water company in England and Wales, raised its leverage from 7% in 1997 to 49% in 2005.\(^1\)

This trend, coined the “dash for debt,” is widespread across countries and across sectors and has raised substantial concerns among policy markers. For instance, a joint study of the UK Department of Trade and Industry (DTI) and the HM Treasury argues that the “dash for debt” within the UK utilities sector from the mid-late 1990’s “could imply greater risks of financial distress, transferring risk to consumers and taxpayers and threatening the future financeability of investment requirements” (DTI and HM Treasury, 2004, p. 6). Moreover, the study argues that “Academic evidence suggests that, for firms in general, increased gearing can lead to a reduction in capital expenditure...” (DTI and HM Treasury, 2004, p. 30). Likewise, the Italian energy regulatory agency, AEEG, has recently expressed its concern that excessive financial leverage could lead to financial distresses which in turn could cause service interruptions (AEEG 2008, paragraph 22.13). The AEEG has also announced its intention to start monitoring the financial leverage of Italian energy utilities in

\(^1\)For more systematic evidence, see Bortolotti et al. (2011) for evidence on the EU14 states and Da Silva et al. (2006) for evidence on Latin America and Asia. Da Silva et al. report that the average market leverage of regulated privatized utilities (net debt divided by net debt plus market value of equity) has increased from 25.4% in 1994 to 59.3% in 2002 in Latin America and from 27.9% in 1994 to 39.2% in 2002 in Asia. The increase is particularly large for electric utilities and for gas distribution companies.
order to discourage speculative behavior that might jeopardize their financial stability (see AEEG, 2007, paragraph 17.40 and AEEG, 2009, paragraph 11.8).

To put the concerns about the dash for debt phenomenon in perspective, it is worth noting that the investments of public utilities in infrastructure account for a significant fraction of GDP. For example, in the EU14 states, the average rate of gross fixed capital formation in the energy sector (electricity and gas), telecommunications, water supply, and transportation, was 15.24% of GDP in 2008 (see Table 1 in the Appendix for details). Given the sheer size of investments at stake and the overall importance of the public utilities sector for the economy at large, it is clearly important to understand the determinants of the investments and financial decisions of regulated firms and study how these decisions affect social welfare.

Existing literature suggests that regulated firms may have an incentive to finance their investments with debt since this induces regulators to raise prices in order to minimize the risk of financial distress (see e.g., Taggart 1981 and 1985; Dasgupta and Nanda, 1993; Spiegel and Spulber, 1994 and 1997; and Spiegel, 1994 and 1996). This literature, however, implicitly assumes that firms are privately owned and regulators are independent. While these assumptions reflect the institutional setting in the U.S., they are inconsistent with the situation in many other countries around the world, including in the EU, Latin America, and Asia, where central or local governments still hold significant ownership stakes (often controlling stakes) in many public utilities (see e.g., Bortolotti and Faccio, 2008; Boubakri and Cosset, 1998; and Boubakri et al., 1998), and IRAs do not exist in all sectors.

The purpose of this paper is to develop a tractable model that will allow us to study the strategic interaction between the capital structure of regulated firms, their investment decisions, and the price setting process, under the explicit assumption that firms are, at least partially, owned by the state and regulators are not fully independent. In particular,

\[2\] Indeed, regulated firms in the U.S. are among the most highly leveraged group of firms: see for example, Bowen, Daly and Huber (1982), Bradley, Jarrell, and Kim (1984), Dasgupta and Nanda (1993) and Barclay, Marx, and Smith (2003).

\[3\] For instance, in the EU, IRAs are fully operational only in the telecommunications and energy sectors, but in other sectors, like transportation and water, most utilities are still regulated directly by ministries, governmental committees, or local governments (see Bortolotti et al. 2011).
we wish to understand how the interaction between capital structure, regulated prices, and the investments is affected by state ownership in the firm, by regulatory independence, and by the regulatory climate (i.e., the degree to which regulators are pro-consumers or pro-firm). Our analysis is motivated in part by the recent empirical evidence in Bortolotti et al. (2011) that the interaction between capital structure and regulation depends critically on the state’s ownership and on regulatory independence. Specifically, they find that regulated firms tend to have a higher leverage when they are privately-controlled and when they are regulated by an IRA. These trends are shown in the following figure that uses the same sample of firms used by Bortolotti et al. (2011):\(^4\)

![Figure 1: The market leverage of 92 publicly traded utilities in the EU14 states (market leverage is defined as total financial debt (both long- and short-term) in book values divided by the sum of total financial debt and the market value of equity). The left panel shows how the existence of an IRA affects leverage, while the right panel shows how ownership structure affects leverage (firms are considered private if the state’s stake in the firm is less than 50%).](image)

Bortolotti et al. (2011) also find that leverage has a positive and significant effect on regulated prices, but not vice versa. Cambini and Rondi (2011) examine a panel of 15

\(^4\)The sample covers 92 publicly traded utilities in the EU14 states over the period 1994-2005. Of the 765 firm-year observations, (i) 464 are on cases where an IRA is in place and 301 are on cases where an IRA is not in place, and (ii) 537 observations are on privately-controlled firms (the government owns more than 50% of the control rights), and 228 are on state-controlled firms. Bortolotti et al. (2011) show that the trends shown in Figure 1 persist even after controlling for various possible determinants of capital structure like firm size, asset tangibility, and non-debt tax shields, as well as for country, sector, and year effects.
EU Public Telecommunication Operators (PTOs) over the period 1994-2005 and find that leverage not only has a positive effect on regulated retail rates, but also on the wholesale charges that alternative operators are required to pay to access the PTOs’ networks. The increased wholesale charges in turn lower the degree of market competition. These findings are consistent with the main premise of this paper, which is that regulated firms use leverage strategically to obtain better regulatory outcomes.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the equilibrium regulated price for given combinations of debt and investment. In Section 4, we solve for the equilibrium choice of capital structure and study how it is affected by the main exogenous parameters of the model, namely the degree regulatory independence, the state’s stake in the regulated firm, and the regulatory climate. In Section 5, we consider the firm’s investment decision and study how it is affected by the main exogenous parameters of the model. In Section 6, we examine the implications of our model for social welfare. Concluding remarks are in Section 7. All proofs are in the Appendix.

2 The model

Consider a regulated firm, which for simplicity (but without a serious loss of insights), faces a unit demand function. The willingness of consumers to pay depends on the firm’s investment, $k$, and is given by a twice differentiable, increasing, and concave function $V(k)$. That is, $k$ can be interpreted as investment in the “quality” of the firm’s services. Using $p$ to denote the regulated price, consumers’ surplus is given by $V(k) - p$.

2.1 The regulated firm’s objective

The regulated firm is partially owned by the state (at the national or the local level). The state’s stake in the firm’s equity is $\delta$. To capture the effect of $\delta$ on the firm’s behavior, we adopt the managerially-oriented public enterprise (MPE) approach, due to Sappington and Sidak’s (2003, 2004). The key assumption in the MPE approach is that the (partially)
state-owned firm is concerned not only with profits, $\pi$, but also with revenues, $R$.\footnote{For related papers in which the effect of state ownership is modelled by modifying the firm’s objective function, see for example, Bös and Peters (1988), De Fraja and Delbono (1989), Fershtman (1990), Cremer, Marchand and Thisse (1989, 1991), and Lee and Hwang (2003).} Ex post, after its investment $k$ is already sunk, the firm’s objective function is

$$\delta R + (1 - \delta) \pi.$$ \hspace{1cm}

Noting that $\pi = R - C$, where $C$ is the firm’s total costs (including its expected cost of financial distress), the objective function of the firm can be written as

$$\delta R + (1 - \delta) (R - C) = R - (1 - \delta) C.$$ \hspace{1cm}

Ex ante, before $k$ is sunk, the firm’s objective is to maximize the expression,

$$\delta R + (1 - \delta) (R - C) - k = R - (1 - \delta) C - k.$$ \hspace{1cm}

The objective function of the partially state-owned regulated firm implies that effectively, the firm ignores a fraction $\delta$ of its cost. This reflects the idea that the managers of state-owned enterprises (and state officials who monitor them) often have considerable interest in expanding the scale or scope of their activities and expand the firm’s budget and its labor force either for political reasons (e.g., cater to the needs of special interest groups), or because they wish to realize the power and prestige that often accompany expanded operations. Alternatively, the objective function can simply reflect managerial slack. While managers of fully private firms may have similar interests, the discipline of capital markets, as well as takeover threats, limit their freedom to pursue their own private agenda. Of course, the managers of partially state-owned firms are also exposed to these forces but to a lesser extent; the objective function captures that idea that the larger is the state’s stake in the firm, the lower is the disciplining force of capital markets, so managers effectively ignore a larger fraction of the firm’s cost.

### 2.2 The capital structure of the firm and its expected cost

The firm’s cost of production is subject to random cost shocks (e.g., fluctuating energy prices) and is given by a random variable, $c$, distributed uniformly over the interval $[0, \bar{c}]$, ...
where $c < V(0)$. Let $D$ denote the face value of the firm’s debt, which the firm needs to cover from its operating income $p - c$. If the firm cannot pay $D$ in full, it incurs a fixed cost $T$ due to financial distress. Using $\phi(p, D)$ to denote the probability of financial distress, the total expected cost of the firm is

$$C = \frac{\bar{c}}{2} + \phi(p, D)T,$$

where

$$\phi(p, D) = \begin{cases} 
0 & D + \bar{c} \leq p, \\
1 - \frac{p-D}{\bar{c}} & D \leq p < \bar{c} + D, \\
1 & p < D.
\end{cases}$$

Intuitively, when $D + \bar{c} \leq p$, the firm can always pay $D$ in full so $\phi(p, D) = 0$. On the other hand, when $p < D$, the firm cannot pay $D$ in full even when $c = 0$, so $\phi(p, D) = 1$. For intermediate cases, $\phi(p, D) = 1 - \frac{p-D}{\bar{c}}$. Obviously, $\phi(p, D)$ is (weakly) increasing with $D$ and (weakly) decreasing with $p$: the firm is more likely to be financially distressed when its debt is high and the regulated price is low.

### 2.3 The rate setting process, regulatory independence, and regulatory climate

Following Dasgupta and Nanda (1993) and Spiegel and Spulber (1997), we assume that the regulator chooses the regulated price, $p$, to maximize a social welfare function defined over consumers’ surplus, $V(k) - p$, and the firm’s objective function.\(^6\) It is often argued that a

\(^6\)Our approach is consistent with the observation that in practice, regulators set prices to balance the interests of consumers and firms. For example, according to the U.S. Supreme Court, price regulation “involves a balancing of the investor’s and the consumers’ interests” that should result in rates “within a range of reasonableness” (see Federal Power Comm. v. Hope Natural Gas Co., 320 U.S. 591, 603 (1944)).Similarly, Ofwat, the water and sewerage regulatory agency in England and Wales states that “...it is our role to protect the interests of consumers while enabling efficient companies to carry out and finance their functions. This is a delicate balancing act. On the one hand, we must be sure that customers continue to receive the services that they expect – at a price they are willing to pay – now and over the long term. On the other, we must ensure that the companies have sufficient resources to deliver services efficiently and remain attractive to investors...” (see Ofwat, 2010, p. 3).
greater degree of regulatory independence improves the regulators’ ability to make long-term commitments to regulatory policies (see e.g., Levy and Spiller, 1994, Gilardi 2002 and 2005, and the discussion in Edwards and Waverman, 2006). In line with this argument, we capture the regulator’s degree of independence by assuming that although the regulator sets $p$ after the firm’s investment, $k$, is sunk, the regulator has some ability to commit to take $k$ into account when setting $p$. Specifically, we assume that before the firm invests, the regulator commits to take into account the ex ante objective function of the firm, $p - (1 - \delta) C - k$, and hence sets $p$ by maximizing the ex ante social welfare function

$$(V(k) - p)^\gamma (p - (1 - \delta) C - k)^{1-\gamma}.$$  

(2)

The parameter $\gamma \in (0, 1)$ captures the regulatory climate: the higher $\gamma$, the more pro-consumer the regulator is. Notice that regulation in our model is characterized by two different parameters: $\rho$ captures the ability of the regulator to make long-term commitments

The regulator keeps this commitment though only with probability $\rho$. With probability $1 - \rho$, the regulator behaves opportunistically and once $k$ is sunk, takes into account the ex post objective function of the firm, $p - (1 - \delta) C$, which does not include $k$. In this case, the regulator chooses $p$ to maximize the ex post social welfare function

$$(V(k) - p)^\gamma (p - (1 - \delta) C)^{1-\gamma}.$$  

(3)

Our model then features regulatory risk in the sense that the firm invests while being uncertain about the exact way the regulated price will set. For recent models of regulatory risk, see Lyon and Li (2004) and Strausz (2011a) and (2011b). In Lyon and Li (2004), the regulator sets $p$ by maximizing a social welfare function similar to the one that we posit, but when the firm invests, it is uncertain if $\gamma = 1$ or $\gamma = 1/2$. Strausz (2011a) studies a model of optimal regulation under the assumption that the firm is uncertain about the weight that the regulator attaches to profits and about the shadow cost of public funds. Strausz (2011b) examines how regulatory risk regrading the weight that the regulator attaches to profits emerges endogenously due electoral competition between two parties with different ideal weights on profits. He studies the implicit incentives of the political system to reduce regulatory risk.
and hence serves as our measure of regulatory independence, with larger values of $\rho$ indicating a greater degree of independence. The parameter $\gamma$ in turn reflects how pro-consumer or pro-firm the regulator is. Although the parameters $\gamma$, $\rho$, and $\delta$ might be correlated (e.g., a more pro-firm regulator may be more committed or a larger stake in the firm may induce the state to lean on the regulator to pursue more pro-firm policies), a-priori we will not impose any restrictions on their relative sizes.

The regulated prices that maximize (2) and (3) allocate the expected social surplus according to the asymmetric Nash bargaining solution for the regulatory process. Under this interpretation, the parameters $\gamma$ and $1 - \gamma$ reflect the bargaining powers of consumers and the firm. Our approach is therefore consistent with models that view the regulatory process as a bargaining problem between consumers and investors (Spulber, 1989; Besanko and Spulber, 1992). Alternatively, the social welfare functions (2) and (3) could also represent a reduced form for the regulator’s own payoff from being involved in some political economy game.

2.4 The sequence of events

The strategic interaction between the firm and the regulator evolves in two stages. In stage 1, the firm chooses $k$ and issues debt with face value $D$ in a competitive capital market. If the funds raised by issuing $D$ exceed $k$, the firm pays the excess funds as a dividend. If the funds raised by issuing $D$ fall short of $k$, the firm raises additional funds by issuing equity; to simplify matters, we assume that in this case the state participates in the equity issue to maintain its original stake $\delta$. In stage 2, given $k$ and $D$, the regulator sets the

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9Our approach differs from De Fraja and Stones (2004) and Stones (2007) where the regulator, rather than the firm, chooses the capital structure of the firm. These paper also assume that the regulator must set $p$ to ensure that the firm never goes bankrupt and shareholders earn their required rate of return. Our approach also differs from Lewis and Sappington (1995) who examine the optimal design of capital structure in the context of an agency model that involves a risk-averse regulator (a principal) and a risk-neutral regulated firm (an agent) under alternative assumptions regarding the principal’s ability to control the agent’s capital structure.

10Without this assumption, there would be another link between the investment decision of the firm, its capital structure, and its ownership structure. However, taking this link into account would require a theory of public ownership (i.e., a theory that would endogenize the state’s stake in the firm). Such a theory is
regulated price \( p \). Finally, the firm’s cost \( c \) is realized, output is produced, and payoffs are realized. Our sequence of events (the firm makes its choices before the regulated price is set) is consistent with the finding in Bortolotti et al. (2011) that leverage Granger causes regulated prices, but not vice versa.

### 3 The regulated price

In stage 2 of the game, the regulator sets \( p \) to maximize either the ex ante social welfare function (2) or the ex post social welfare function (3). Since the two welfare functions differ only with respect to whether \( k \) is taken into account, we can rewrite the regulator’s objective function compactly as

\[
(V(k) - p)^\gamma (p - (1 - \delta) C - Ik)^{1-\gamma},
\]

where \( I \) is an indicator function which equals 1 with probability \( \rho \) (the regulator keeps his commitment to take \( k \) into account) and equals 0 with probability \( 1 - \rho \) (the regulator behaves opportunistically and ignores \( k \) when he sets \( p \)). Using (4), we can now solve the problems of both committed and opportunistic regulators by simply maximizing (4) with respect to \( p \). Using the same steps as in Spiegel (1994), the solution to the maximization problem is given by

\[
p^*(D, k, I) = \begin{cases} 
D_1(k, I) + \bar{c} & D \leq D_1(k, I), \\
D + \bar{c} & D_1(k, I) < D \leq D_2(k, I), \\
D_1(k, I) + \bar{c} + M(D, I) & D_2(k, I) < D \leq D_3(k, I), \\
D_1(k, I) + \bar{c} + \gamma (1 - \delta) T & D > D_3(k, I),
\end{cases}
\]

where

\[
D_1(k, I) \equiv (1 - \gamma) V(k) + \gamma (1 - \delta) \bar{c} / 2 + \gamma Ik - \bar{c},
\]

\[
M(D, I) \equiv \frac{\gamma (1 - \delta) T}{1 + (1 - \delta) T} \left( D + (1 + \delta) \frac{\bar{c}}{2} - Ik \right),
\]

\[
D_2(k, I) \equiv \frac{D_1(k, I) \left( 1 + (1 - \delta) \frac{T}{T} \right) + \gamma (1 - \delta) T \left( (1 + \delta) \frac{\bar{c}}{2} - Ik \right)}{1 + (1 - \gamma) (1 - \delta) T / \bar{c}},
\]

beyond the scope of the current paper.
and $D_3(k, I)$ is smaller than the value of $D$ for which $D_1(k, I) + \bar{c} + M(D, I) = D$. This solution is obtained under the assumption that $\gamma < \frac{V(0) - \pi}{V(0) - (1 - \delta)^{\frac{\alpha}{2}}}$ (the regulator is not too pro-consumer). If this assumption is violated, then $D_1(k, 0) = 0$, though none of our results is affected. The regulated price is illustrated in the following figure:

![Illustrating the regulated price as a function of D for I = 0 (the solid red line) and I = 1 (the dashed blue line), holding k fixed](image)

Figure 2: Illustrating the regulated price as a function of $D$ for $I = 0$ (the solid red line) and $I = 1$ (the dashed blue line), holding $k$ fixed

To interpret Figure 2, note that if we ignore financial distress, i.e., assume that $\phi(p, D) = 0$, then the price that maximizes (4) is given by $D_1(k, I) + \bar{c}$. So long as $D \leq D_1(k, I)$, this price covers the firm’s cost plus its debt obligation even in the worst state of nature.\(^{11}\) Hence, $\phi(p, D)$ is indeed equal to 0 for all $D \leq D_1(k, I)$. However, once $D > D_1(k, I)$, a price of $D_1(k, I) + \bar{c}$ leaves the firm susceptible to financial distress. So long as $D$ does not exceed $D_1(k, I)$ by too much, the regulator finds it optimal to set $p = D + \bar{c}$ to keep $\phi(p, D)$ just equal to 0. However, when $D > D_2(k, I)$, this strategy is no longer optimal for the regulator because the resulting marginal loss in consumers’ surplus becomes too large relative to the benefit of preventing financial distress. Therefore, although the regulator continues to increase $p$ with $D$, the slope is now below 1 and the resulting $p$ is

\(^{11}\)As mentioned above, if $\gamma$ is relatively large, then $D_1(k, I) = 0$ and the regulator cannot ignore the possibility of financial distress, no matter how small $D$ is.
smaller than $D + \tau$; hence now $\phi(p, D) > 0$. When $D > D_3(k, I)$, it is no longer optimal for the regulator to offset the effect of debt on the likelihood of financial distress. Consequently, $\phi(p, D) = 1$, and therefore $p$ is now constant and equals $D_1(k, I) + \tau + (1 - \delta)T$.

It is easy to see from equations (6) and (8) that $D_1(k, 1) > D_1(k, 0)$ and $D_2(k, 1) > D_2(k, 0)$, and moreover, it is easy to check from (5) that $p^*(D, k, 1) \geq p^*(D, k, 0)$: the regulated price set by a committed regulator (who takes $k$ into account) is weakly higher than price set by an opportunistic regulator (who ignores $k$). To limit the number of different cases that can arise, we make the following assumption:

**Assumption 1:** $D_1(k, 1) < D_2(k, 0)$.

Assumption 1 ensures that the parameters of the model are such that there exists an interval of $D$ for which $p^*(D, k, 1) = p^*(D, k, 0)$.\footnote{Absent Assumption 1, $p^*(D, k, 1) > p^*(D, k, 0)$ for all $D$, although none of our main results is affected.} A sufficient condition for Assumption 1 to hold is that the surplus from investment, $V(k) - k$, is sufficiently large:

$$V(k) - k > \frac{k}{(1 - \gamma)(1 - \delta)} + (1 - \delta) \frac{\tau}{\sigma}.$$ 

Assumption 1, together with the fact that $D_2(k, 0) < D_2(k, 1)$, implies that, as Figure 2 shows,

$$D_1(k, 0) < D_1(k, 1) < D_2(k, 0) < D_2(k, 1).$$

## 4 The choice of capital structure

Assuming that the capital market is perfectly competitive, the market value of new equity and debt is exactly equal in equilibrium to their expected return. Hence, outside investors (debtholders and possibly new equityholders if the firm also issues new equity) must break even. This implies in turn that the entire expected profit of the firm, $p - C$, net of the sunk cost of investment, $k$, must accrue to the original equityholders.

To write down the firm’s objective function, let $\phi^*(D, k, I) \equiv \phi^*(p^*(D, k, I), D)$ be the probability of financial distress, which is obtained by substituting $p^*(D, k, I)$ into equation (1). The expected cost of the firm is then $C = \frac{\tau}{2} + \phi^*(D, k, I)T$. Now, recall that with
probability \( \rho \), the regulator is committed to take \( k \) into account, in which case the regulated price is \( p^*(D, k, 1) \) and the probability of financial distress is \( \phi^*(D, k, 1) \). With probability \( 1 - \rho \), the regulator is opportunistic, so the regulated price and probability of financial distress are \( p^*(D, k, 0) \) and \( \phi^*(D, k, 0) \). Since the original equityholders ignore a fraction \( \delta \) of the firm’s cost, their expected payoff is equal to

\[
Y(D, k) = \rho \left[ p^*(D, k, 1) - (1 - \delta) \left( \frac{\bar{c}}{2} + \phi^*(D, k, 1) T \right) - k \right] + (1 - \rho) \left[ p^*(D, k, 0) - (1 - \delta) \left( \frac{\bar{c}}{2} + \phi^*(D, k, 0) T \right) - k \right]. \tag{9}
\]

The firm chooses its debt level, \( D \), and investment, \( k \), to maximize \( Y(D, k) \). The following proposition characterizes the equilibrium choice of debt. The proof, as well as all other proofs, is in the Appendix.

**Proposition 1:** In equilibrium, the regulated firm will issue debt with face value \( D_2(k, 0) \) if \( \rho < \rho^* \), and will issue debt with face value \( D_2(k, 1) \) if \( \rho > \rho^* \), where

\[
\rho^* \equiv \frac{(1 - \gamma)(1 - \delta) \frac{T}{r}}{1 + (1 - \gamma)(1 - \delta) \frac{T}{r}}. \tag{10}
\]

Proposition 1 shows that the capital structure of the firm depends on \( \rho \), which reflects the degree of regulatory independence. In what follows, we will say that the regulator is “independent” if \( \rho > \rho^* \) (the regulator’s ability to commit to take \( k \) into account is relatively high) and “non independent” if \( \rho < \rho^* \) (the regulator’s ability to commit is relatively low). Proposition 1 shows that the firm issues more debt when it faces an independent regulator. Note from (10) that the threshold \( \rho^* \) above which we consider the regulator as “independent” is decreasing with both \( \gamma \) and \( \delta \): other things equal, a more pro-consumer regulator (a higher \( \gamma \)) who faces a less privatized firm (a higher \( \delta \)) is considered “independent” for a larger range of values of \( \rho \).

We now establish two corollaries to Proposition 1.

**Corollary 1:** When the regulator is non independent (\( \rho < \rho^* \)), the regulated price is equal to \( D_2(k, 0) + \bar{c} \) with probability 1. When the regulator is independent (\( \rho > \rho^* \)), the regulated price is equal to \( D_2(k, 1) + \bar{c} \) with probability \( \rho \) and \( D_1(k, 0) + \bar{c} + M(D_2(k, 1), 0) \) with
probability $1 - \rho$, where $D_2(k, 1) + \bar{c} > D_1(k, 0) + \bar{c} + M(D_2(k, 1), 0)$. The expected regulated price when $\rho > \rho^*$ is therefore

$$Ep^*(k) = \rho D_2(k, 1) + (1 - \rho) (D_1(k, 0) + M(D_2(k, 1), 0)) + \bar{c}. \quad (11)$$

Corollary 1 shows that, counterintuitively, the regulated price can be fully anticipated only when the regulator is non-independent. This result is counterintuitive because an independent regulator has a greater ability to commit to the way the regulated price will be set. However, precisely for this reason, the regulated firm issues in this case debt with a larger face value. This debt level in turn induces the regulator to set a lower price when he happens to be opportunistic than he would if he happens to be committed.

The next corollary deals with financial distress. When the regulator is non-independent ($\rho < \rho^*$), the firm issues debt with face value $D_2(k, 0)$. Since by Corollary 1, the resulting regulated price is $D_2(k, 0) + \bar{c}$, the firm is immune to financial distress even when the highest cost shock is realized. When the regulator is independent ($\rho > \rho^*$), the firm’s debt is $D_2(k, 1)$. By Corollary 1, the regulated price in this case is $D_2(k, 1) + \bar{c}$; with probability $\rho$, this price ensures once again that the firm never becomes financially distressed. With probability $1 - \rho$, though, the regulated price is $D_1(k, 0) + \bar{c} + M(D_2(k, 1), 0)$; since this price is below $D_2(k, 1) + \bar{c}$, the firm now becomes financially distressed when the cost shock is sufficiently large.

**Corollary 2:** When the regulator is non-independent ($\rho < \rho^*$), the firm is completely immune to financial distress. When the regulator is independent ($\rho > \rho^*$), the firm is immune to financial distress with probability $\rho$ (the regulator is committed); with probability $1 - \rho$ (the regulator is opportunistic), the firm becomes financially distressed when $c$ is sufficiently high.

Corollary 2 shows another counterintuitive implication of Proposition 1: the regulated firm may become financially distressed only when the regulator is independent. As before, the reason is that in this case, the firm allows itself to issue debt with a higher face value. With probability $1 - \rho$, the regulator happens to be opportunistic, and sets a regulated price that leaves the firm susceptible to financial distress with a positive probability.
With Proposition 1 in place, we can now examine how the equilibrium debt level is affected by the main exogenous parameters of the model, holding the firm’s investment level, \( k \), fixed. Proposition 1 already shows that the firm will issue more debt when the regulator is independent \((\rho > \rho^*)\) than when the regulator is non independent \((\rho < \rho^*)\). In the next proposition, we examine how debt is affected by the other two main exogenous parameters: the state’s stake in the regulated firm, \( \delta \), and the measure of regulatory climate (i.e., how pro-consumer the regulator is), \( \gamma \).

**Proposition 2:** Holding \( k \) fixed, the debt level of the regulated firm is higher the lower \( \delta \) and \( \gamma \) are.

Combined, Propositions 1 and 2 imply that if we consider a cross section of regulated firms that differ in terms of the degree to which they are privatized (the value of \( \delta \)) and in terms of the regulatory environment they operate in (the values of \( \rho \) and \( \gamma \)), then other things equal, firms that are more privatized (\( \delta \) is lower) and face more independent and more pro-firm regulators (\( \rho \) is higher and \( \gamma \) is lower) should be more leveraged. These predictions are consistent with Bortolotti et al. (2011) who study a comprehensive panel data of 92 publicly traded EU utilities over the period 1994–2005 and find that firms tend to be more leveraged if they are privately controlled (i.e., the state’s stake in the firm is below 50% or below 30%) and regulated by an independent regulatory agency.\(^{13}\) Although Bortolotti et al. establish their results without controlling for investments, we show in Proposition 7 below that the predictions of Propositions 1 and 2 generalize to the case where \( k \) is determined endogenously.

To see the intuition for Proposition 2, note that in equilibrium, the firm issues the largest \( D \) that still ensures that if the regulator is committed, the firm will be completely immune to financial distress. Naturally then, the firm will issue a higher \( D \) if \( p \) is higher. When the state holds a smaller stake in the firm, the firm takes into account a larger fraction of its cost, so the regulator, who sets \( p \) by taking into account the firm’s objective function,

\(^{13}\)Bortolotti et al. (2011) do not have a direct measure of the regulatory climate and hence cannot study the effect of the regulatory climate on leverage and on prices. Their analysis shows however that firms have a lower leverage when the government is more right-wing. To the extent that right-wing governments are more pro-firm, this finding is inconsistent with Proposition 2.
will set a higher $p$. Likewise, given $D$, $p$ is higher when the regulator is more pro-firm. In both cases, the fact that $p$ is higher allows the firm to issue a higher $D$. Since other things equal, $p$ is higher when the regulator is independent, the firm will also issue a higher $D$ when it faces an independent regulator.

Next, we examine how the regulated price is affected by $\delta$ and $\gamma$. As in the case of Proposition 2, we hold $k$ fixed for the moment; in Section 5, we will consider the endogenous determination of $k$ and show that our comparative statics results continue to hold.

**Proposition 3:** Holding $k$ fixed, the expected regulated price is higher when the regulator is independent ($\rho > \rho^*$) than it is when the regulator is non independent ($\rho < \rho^*$). Moreover, the expected regulated price is decreasing with both the state’s ownership stake $\delta$, and with the measure of regulatory climate $\gamma$.

Combined, Propositions 2 and 3 imply that if we hold $k$ fixed, then any change in the parameters $\rho$, $\delta$ and $\gamma$ shifts the firm’s debt and the regulated price in the same direction. This implies in turn that in a sample of regulated firms that differ from each other only in terms of $\rho$, $\delta$ and $\gamma$, the firm’s debt and regulated price should be positively correlated.

Finally, recall from Corollary 2 that the firm never becomes distressed if $\rho < \rho^*$. When $\rho > \rho^*$, the firm becomes distressed only when the regulator is opportunistic and sets a regulated price equal to $p^* (D_2(k, 1), k, 0) = D_1(k, 0) + \bar{c} + M (D_2(k, 1), 0)$. Since the probability of this event is $1 - \rho$, the overall probability of financial distress when $\rho > \rho^*$ is $(1 - \rho) \phi^I (k)$, where, using equation (1),

$$
\phi^I (k) \equiv 1 - \frac{p^* (D_2(k, 1), k, 0) - D_2(k, 1)}{\bar{c}} \phi^*(D_2(k, 1), k, 0)
= \frac{D_2(k, 1) - D_1(k, 0) - M (D_2(k, 1), 0)}{\bar{c}}
= \frac{\gamma k}{\bar{c}} \frac{1}{(1 + (1 - \delta) \frac{\bar{c}}{\gamma})}.
$$

The following result is an immediate consequence of equation (12):

**Proposition 4:** Holding $k$ fixed, the probability of financial distress when an independent regulator happens to be opportunistic, $\phi^I (k)$, is increasing with $\delta$, $\gamma$, and $k$ and is independent of $\rho$. Under a non-independent regulator, the firm never becomes financially distressed.
At a first glance, Proposition 4 seems counterintuitive since Proposition 2 implies that the firm issues a smaller debt, $D$, when $\delta$ and $\gamma$ are higher. Hence it might be thought that the firm would be less susceptible to financial distress. Yet, Proposition 3 shows that when $\delta$ and $\gamma$ are higher, the regulated price, $p$, is also lower. It turns out that the decrease in $p$ has a stronger effect on the probability of financial distress than the decrease in $D$, so overall, financial distress becomes more likely.

5 The equilibrium level of investment

Having characterized the equilibrium choice of debt, we next turn to the choice of investment. Consider first the case where $\rho < \rho^*$, and recall from Corollaries 1 and 2 that in this case, $D = D_2(k, 0)$. The regulator in turn sets a price $D_2(k, 0) + \bar{c}$ which ensures that the firm is completely immune to financial distress. By equation (9) then, the resulting expected payoff of the firm is

$$Y^{NI}(k) \equiv Y(D_2(k, 0), k) = D_2(k, 0) + (1 + \delta) \frac{\bar{c}}{2} - k. \quad (13)$$

When $\rho > \rho^*$, the firm issues debt with face value $D_2(k, 1)$. Now, with probability $\rho$, the regulator is committed and sets a regulated price $p^*(D_2(k, 1), k, 1) = D_2(k, 1) + \bar{c}$, which ensures that the firm never becomes financially distressed. With probability $1 - \rho$, the regulator is opportunistic and sets a regulated price of $p^*(D_2(k, 1), k, 0) = D_1(k, 0) + \bar{c} + M(D_2(k, 1), 0)$; with this price, the firm becomes financially distressed with probability $\phi^I(k)$. Substituting these expressions in equation (9), using the definition of $M(D_2(k, 1), 0)$, and rearranging terms (see the proof of Proposition 5 for details), the firm’s expected payoff is

$$Y^I(k) \equiv Y(D_2(k, 1), k) = (1 - \gamma (1 - \rho^*)) V(k) - (1 - \gamma (\rho - \rho^*)) k \quad (14)$$

$$+ \frac{(1 - \gamma) \left(1 + (1 - \delta) \frac{T}{r}\right) \bar{c}}{1 + (1 - \gamma) (1 - \delta) \frac{T}{r}}.$$  

Using $Y^{NI}(k)$ and $Y^I(k)$ we establish the following result:

**Proposition 5:** The equilibrium level of investment, $k^*$, is independent of the degree of regulatory independence, $\rho$, when $\rho < \rho^*$, but is increasing with $\rho$ when $\rho > \rho^*$. Consequently,
the firm invests more when the regulator is independent (i.e., $\rho > \rho^*$) than when the regulator is non independent (i.e., $\rho < \rho^*$).

Having fully characterized $k^*$ and showed how it is affected by regulatory independence, we are now ready to examine how $k^*$ is affected by the state’s stake in the firm, $\delta$, and by the regulatory climate, $\gamma$, which reflects the degree to which the regulator is pro-consumers.

**Proposition 6:** The equilibrium level of investment, $k^*$, is decreasing with $\delta$ and $\gamma$. If in addition $\frac{V''(k)}{V''(k)}$ is nondecreasing, then the negative effects of $\delta$ and $\gamma$ on $k^*$ are larger when the regulator is independent, i.e., when $\rho > \rho^*$.

To see the intuition for Proposition 6, recall from Proposition 2 that when $\delta$ and $\gamma$ are higher, the regulator sets a lower regulated price. Consequently, the marginal benefit of investment falls and the firm invests less. Proposition 6 shows that these effects are stronger when the regulator is independent, i.e., when $\rho > \rho^*$. Proposition 6 implies that other things equal, firms should invest less when they are less privatized (i.e., $\delta$ is higher), and when they face a more pro-consumer regulator (i.e., $\gamma$ is higher), especially if the regulator is independent.

Propositions 5 and 6 are consistent with existing empirical evidence. Wallsten (2001) studies the investment of Telecoms in 30 African and Latin American countries from 1984 to 1997. Among other things, he finds that privatization combined with regulatory independence is positively correlated with investment in capacity and phone penetration. Privatization alone, however, is associated with few benefits, and is negatively correlated with interconnection capacity. Henisz and Zelner (2001) study data from 55 countries over 20 years and find that stronger constraints on executive discretion, which improves their ability to commit not to expropriate the property of privately owned regulated firms, leads to a faster deployment of basic telecommunications infrastructure. Gutiérrez (2003) examines how regulatory governance affected the performance of telecoms in 22 Latin American countries during the period 1980–1997 and finds that regulatory independence has a positive impact on network expansion and efficiency. Alesina et al. (2005) examine the aggregate levels of investment in the transport, telecommunications, and energy sectors in 21 OECD
countries over the period 1975-1998. Among other things, they show that a larger ownership stake of the state is associated with lower levels of investment. Egert (2009) shows that incentive regulation implemented jointly with an independent sector regulator has a strong positive impact on investment in various network industries (electricity, gas, water supply, road, rail, air transportation, and telecommunications) in OECD member countries. Finally, Cambini and Rondi (2010) study a panel of 80 publicly traded EU telecoms, energy, transportation, and water utilities over the 1994-2004 period and find that utilities invest more when an IRA is in place; moreover, they find that conditional on the existence of an IRA, firms invest more when the IRA has a larger degree of formal independence.

Next, recall that Propositions 1-4 examined the effects of regulatory independence, regulatory climate, and ownership structure on the firm’s debt level, regulated price, and the probability of distress, holding $k$ fixed. We now show that these results continue to hold even after the endogenous choice of $k$ is taken into account.

**Proposition 7:** Taking into account the endogenous choice of investment, the firm’s debt and the regulated price are higher when $\rho > \rho^*$ (the regulator is independent) than they are when $\rho < \rho^*$ (the regulator is non-independent). Moreover, the firm’s debt and the regulated price are both decreasing with the state’s ownership stake $\delta$, and with the measure of regulatory climate $\gamma$. The probability of financial distress when an independent regulator is opportunistic, $\phi^I (k^*)$, is increasing with the degree of regulatory independence, $\rho$. If in addition $\gamma$ is sufficiently small to ensure that $\frac{\nu''(k^*)}{\nu''(k^*)k^*} + \frac{(1-\gamma)(1+(1-\gamma)(1-\delta))}{\gamma} \geq 0$, then $\phi^I (k^*)$ is also increasing with the state’s ownership stake, $\delta$, and with the measure of regulatory climate, $\gamma$.

The result that $\phi^I (k^*)$ is increasing with the degree of independence, $\rho$, is surprising given that an increase in $\rho$ means that the regulator is less likely to be opportunistic (recall that financial distress occurs only when the regulator is opportunistic). The reason for this surprising result is that when the regulator is independent, an increase in $\rho$ induces the firm to invest more and to issue more debt to finance its investment. Indeed, Proposition 4 shows that $\phi^I (k)$ is increasing with $k$ and Proposition 5 shows that $k^*$ is increasing with $\rho$. As a result, an increase in $\rho$ makes the firm more susceptible to financial distress. Proposition
7 also shows that the result of Proposition 4 that the firm is more susceptible to financial distress as $\delta$ and $\gamma$ increase continues to hold when $k$ is endogenous, provided that $\gamma$ is sufficiently low.

To get a better feel for the sufficient condition in the last part of Proposition 7, suppose that $V (k) = \log (a + k)$, where $a < 1$. Then, $\frac{V'(k^*)}{V''(k^*)k^*} = -\frac{1}{\frac{a}{a+k^*}} = -(1 + \frac{a}{k^*})$. In the proof of Proposition 8 below we show that $V'(k^*) > 1$. In the current example, this inequality implies that $\frac{1}{a+k^*} > 1$, or $k^* < 1 - a$. Hence, $\frac{V'(k^*)}{V''(k^*)k^*} = -(1 + \frac{a}{k^*}) < -\frac{1}{1-a}$. The sufficient condition then is more likely to hold as $a$ gets smaller.

6 Social welfare

Having studied the firm’s investment and financing decisions, we now turn to the implications of our model for social welfare. In particular, we are interested in finding out how regulatory independence, state ownership in the firm, and the regulatory climate affect social welfare once the firm’s and the regulator’s decisions are taken into account. In our model, the expected value of social welfare is given by the difference between the willingness of consumers to pay and the expected cost of the firm, including its expected cost of financial distress and cost of investment:

$$W (k) = V (k) - \frac{c}{2} - (1 - \rho) \phi^* (D, k, I) T - k.$$  

By Corollary 2, $\phi^* (D, k, I) = 0$ when the regulator is not independent. Hence, the expected social welfare, as a function of $k$, is given in this case by

$$W^{NI} (k) = V (k) - \frac{c}{2} - k. \quad (15)$$

When the regulator is independent, equation (12) shows that $\phi^* (D, k, I) = \frac{\gamma k}{c(1+(1-\delta)^{\frac{T}{\delta}})}$. Hence, expected social welfare, as a function of $k$, is given by

$$W^I (k) = V (k) - \frac{c}{2} - \frac{(1 - \rho) \gamma k T}{1 + (1 - \delta) \frac{T}{\delta}} - k. \quad (16)$$

In the next proposition we compare the equilibrium level of investment, $k^*$, with the socially optimal level that maximizes $W^{NI} (k)$ and $W^I (k)$.
Proposition 8: The equilibrium level of investment, $k^*$, is lower than the socially optimal level. Moreover, in equilibrium,

(i) social welfare is independent of the degree of regulatory independence, $\rho$, but is decreasing with the state’s ownership stake $\delta$, and with the measure of regulatory climate $\gamma$ when the regulator is non-independent (i.e., when $\rho < \rho^*$);

(ii) assuming that $1 - \delta (1 - \gamma) \frac{T}{\pi} > 0$, social welfare is increasing with the degree of regulatory independence, $\rho$, and decreasing with the state’s ownership stake $\delta$, and with the measure of regulatory climate $\gamma$, when the regulator is independent (i.e., when $\rho > \rho^*$).

Proposition 8 shows that when we take into account the endogenous determination of investment and capital structure, a higher degree of regulatory independence (a higher $\rho$), a larger extent of privatization (a decrease in the value of $\delta$), and a more pro-firm regulatory climate (a lower value of $\gamma$), are all welfare-enhancing. The reason for this is that as Propositions 5-6 show, regulatory independence, privatization and pro-firm regulatory climate strengthen the firm’s incentive to invest and this increases the total surplus generated by the firm.

7 Conclusion

We studied the strategic interaction between capital structure, regulation, and investment, in a setting that features partial ownership by the state in the regulated firm and regulation by agencies with various degrees of independence. Both features are common in many countries around the world. Our model shows that regulated firms increase issue more debt, invest more, and enjoy higher regulated prices when they face independent regulators, when the state owns a smaller fraction of the firm, and when regulators are more pro-firm. Moreover, regulatory independence, higher degree of privatization (the state’s stake in the firm is smaller), and pro-firm regulatory climate are all welfare-enhancing.

Our results indicate that the “dash for debt” phenomenon observed in many countries is a natural response of regulated utilities to the privatization process and the establishment
of independent regulatory agencies. Our results also indicate that while the increase in debt is associated with higher regulated prices, it is also associated with more investment, and more importantly, higher social surplus.
8 Appendix

Investment rate of utilities relative to GDP in the EU14 states:

The following table shows the rate of gross fixed capital formation in the energy sector (electricity and gas), water supply, transport, and telecommunications, as a share of GDP in 2008, using the OECD’s STAN (Structural Analysis) Indicators database. This database provides annual sectorial indicators on the production and employment structures, labor productivity and costs, investments, R&D expenditures, and international trade patterns in each OECD country.

<table>
<thead>
<tr>
<th>State</th>
<th>Investment rate as % of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>13.94%</td>
</tr>
<tr>
<td>Belgium</td>
<td>15.57%</td>
</tr>
<tr>
<td>Denmark</td>
<td>18.80%</td>
</tr>
<tr>
<td>Finland</td>
<td>15.79%</td>
</tr>
<tr>
<td>France</td>
<td>9.84%</td>
</tr>
<tr>
<td>Germany</td>
<td>11.70%</td>
</tr>
<tr>
<td>Greece</td>
<td>14.59%</td>
</tr>
<tr>
<td>Ireland</td>
<td>19.00%</td>
</tr>
<tr>
<td>Italy</td>
<td>16.63%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>9.66%</td>
</tr>
<tr>
<td>Portugal</td>
<td>20.24%</td>
</tr>
<tr>
<td>Spain</td>
<td>14.58%</td>
</tr>
<tr>
<td>Sweden</td>
<td>18.51%</td>
</tr>
<tr>
<td>UK</td>
<td>14.47%</td>
</tr>
<tr>
<td>Average EU14</td>
<td>15.24%</td>
</tr>
</tbody>
</table>
Proof of Proposition 1: Differentiating equation (9) yields

\[
\frac{\partial Y(D, k)}{\partial D} = \rho \left[ \frac{\partial p^* (D, k, 1)}{\partial D} - (1 - \delta) \left( \frac{\partial \phi^* (D, k, 1)}{\partial p^*} \frac{\partial p^* (D, k, 1)}{\partial D} + \frac{\partial \phi^* (D, k, 1)}{\partial D} \right) \right] T
\]

\[+ (1 - \rho) \left[ \frac{\partial p^* (D, k, 0)}{\partial D} - (1 - \delta) \left( \frac{\partial \phi^* (D, k, 0)}{\partial p^*} \frac{\partial p^* (D, k, 0)}{\partial D} + \frac{\partial \phi^* (D, k, 0)}{\partial D} \right) \right] T. \quad (17)
\]

Note first that when \( D \leq D_2(k, 0) \), \( \phi^*(D, k, 0) = \phi^*(D, k, 1) = 0 \), while \( \frac{\partial p^*(D, k, 0)}{\partial D} \geq 0 \) and \( \frac{\partial p^*(D, k, 1)}{\partial D} \geq 0 \). Hence, \( \frac{\partial Y(D, k)}{\partial D} \geq 0 \) for all \( D \leq D_2(k, 0) \), implying that the firm’s debt will be at least \( D_2(k, 0) \).

Second, consider the range where \( D_2(k, 1) < D < D_3(k, 0) \). Here, \( p^* (D, k, I) = D_1 (k, I) + \bar{c} + M(D, I) \) and \( \phi^* (D, k, I) = 1 - \frac{p^*(D, k, I) - D}{\bar{c}} \). Hence,

\[
\frac{\partial p^* (D, k, I)}{\partial D} = \frac{\partial M (D, I)}{\partial D} = \frac{\gamma (1 - \delta) T}{\bar{c} + (1 - \delta) T},
\]

and

\[
\frac{\partial \phi^* (D, k, I)}{\partial p^*} = - \frac{\partial \phi^* (D, k, I)}{\partial D} = - \frac{1}{\bar{c}}.
\]

Substituting in (17), yields

\[
\frac{\partial Y(D, k)}{\partial D} = \frac{\gamma (1 - \delta) T}{1 + (1 - \delta) \frac{T}{\bar{c}}} - (1 - \delta) \left( 1 - \frac{\gamma (1 - \delta) T}{1 + (1 - \delta) \frac{T}{\bar{c}}} \right) \frac{T}{\bar{c}}
\]

\[= - (1 - \gamma) (1 - \delta) \frac{T}{\bar{c}} < 0.
\]

Moreover, it is easy to see from equation (5) and Figure 2 that \( p^* (D, k, I) \) jumps downward at \( D = D_3(k, 0) \) and is independent of \( D \) for all \( D > D_3(k, 0) \). Hence, \( \frac{\partial Y(D, k)}{\partial D} < 0 \) for all \( D \geq D_2(k, 1) \), implying that the firm will never issue debt with face value above \( D_2(k, 1) \).

Finally, we need to consider the range where \( D_2(k, 0) \leq D \leq D_2(k, 1) \). Figure 2 shows that in this range \( p^* (D, k, 1) = D + \bar{c} \), and \( p^* (D, k, 0) = D_1 (k, 0) + \bar{c} + M(D, 0) \). Hence, \( \phi^* (D, k, 1) = 0 \) and \( \phi^* (D, k, 0) = 1 - \frac{p^*(D, k, 0) - D}{\bar{c}} \). Noting that \( \frac{\partial p^*(D, k, 1)}{\partial D} = 1 \),

\[
\frac{\partial p^* (D, k, 0)}{\partial D} = \frac{\partial M (D, 0)}{\partial D} = \frac{\gamma (1 - \delta) T}{\bar{c} + (1 - \delta) T},
\]

and

\[
\frac{\partial \phi^* (D, k, 0)}{\partial p^*} = - \frac{\partial \phi^* (D, k, 0)}{\partial D} = - \frac{1}{\bar{c}}.
\]
Substituting in (17), yields
\[
\frac{\partial Y(D, k)}{\partial D} = \rho + (1 - \rho) \left[ \frac{\gamma (1 - \delta) T}{1 + (1 - \delta) \frac{T}{c}} - (1 - \delta) \left( 1 - \frac{\gamma (1 - \delta) T}{1 + (1 - \delta) \frac{T}{c}} \right) \frac{T}{c} \right] \\
= \rho - (1 - \rho)(1 - \gamma)(1 - \delta) \frac{T}{c} \\
= \left( 1 + (1 - \gamma)(1 - \delta) \frac{T}{c} \right) \left[ \rho - \frac{(1 - \gamma)(1 - \delta) \frac{T}{c}}{1 + (1 - \gamma)(1 - \delta) \frac{T}{c}} \right] .
\]

If \( \rho < \rho^* \), then \( \frac{\partial Y(D, k)}{\partial D} < 0 \), so the firm will set \( D = D_2(k, 0) \). If \( \rho > \rho^* \), then \( \frac{\partial Y(D, k)}{\partial D} > 0 \), so the firm will set \( D = D_2(k, 1) \). □

**Proof of Corollary 1:** When \( \rho < \rho^* \), the firm issues debt with fact value \( D_2(k, 0) \). By (5),
\[
p^*(D, k, 1) = p^*(D, k, 0) = D_2(k, 0) + \bar{c}.
\]
That is, the regulated price is the same irrespective of whether the regulator is committed or opportunistic.

When \( \rho > \rho^* \), the firm issues debt with face value \( D = D_2(k, 1) \). By (5), the regulated price under a committed regulator is
\[
p^*(D_2(k, 1), k, 1) = D_2(k, 1) + \bar{c},
\]
while the price under an opportunistic regulator is
\[
p^*(D_2(k, 1), k, 0) = D_1(k, 0) + \bar{c} + M(D_2(k, 1), 0) .
\]
The expected price is then given by (11). Noting from Figure 2 that
\[
D_2(k, 1) + \bar{c} > D_1(k, 0) + \bar{c} + M(D_2(k, 1), 0) ,
\]
it follows that \( p^*(D_2(k, 1), k, 1) > p^*(D_2(k, 1), k, 0) \): the price is higher when the regulator is committed. □

**Proof of Proposition 2:** Differentiating \( D_2(k, I) \) with respect to \( \delta \) and \( \gamma \), yields:
\[
\frac{\partial D_2(k, I)}{\partial \delta} = -\gamma \left( (1 - \gamma) (V(k) - I) \frac{T}{c} + \frac{\bar{r}}{2} \right) \left( 1 + (1 - \gamma)(1 - \delta) \frac{T}{c} \right)^2 < 0 , \tag{20}
\]

25
and
\[
\frac{\partial D_2(k, I)}{\partial \gamma} = -\frac{(1 + (1 - \delta) \frac{T}{\delta}) (V(k) - (1 - \delta) \frac{T}{\delta} - Ik)}{(1 + (1 - \gamma) (1 - \delta) \frac{T}{\delta})^2} < 0, \tag{21}
\]
where the inequalities follow by Assumption 1. ■

**Proof of Proposition 3:** First, suppose that \( \rho < \rho^* \). By Corollary 1, the regulated price is then \( D_2(k, 0) + \bar{\tau} \). Since Proposition 2 shows that \( D_2(k, 0) \) decreases with \( \delta \) and \( \gamma \), so does the regulated price.

Second, suppose that \( \rho > \rho^* \). As Corollary 1 shows, the regulated price is then equal to \( D_2(k, 1) + \bar{\tau} \) with probability \( \rho \) and to \( D_1(k, 0) + \bar{\tau} + M(D_2(k, 1), 0) \) with probability \( 1 - \rho \), and the expected regulated price, \( Ep^*(k) \), is given by (11). It is easy to see from Figure 2 that
\[
D_2(k, 1) + \bar{\tau} > D_1(k, 0) + \bar{\tau} + M(D_2(k, 1), 0) > D_2(k, 0) + \bar{\tau}.
\]
Hence, \( Ep^*(k) > D_2(k, 0) + \bar{\tau} \), implying that if we hold \( k \) fixed, the regulated price is higher in expectation when the regulator is independent than when he is not.

Using (11) along with equations (6) and (7), and using (20) and (21), yields
\[
\frac{\partial Ep^*(k)}{\partial \delta} = \left( \rho + (1 - \rho) \frac{\gamma (1 - \delta) \frac{T}{\delta}}{1 + (1 - \delta) \frac{T}{\delta}} \right) \frac{\partial D_2(k, 1)}{\partial \delta} - (1 - \rho) \frac{\gamma (\frac{T}{\delta} + (\bar{\tau} + D_2(k, 1)) \frac{T}{\delta})}{(1 + (1 - \delta) \frac{T}{\delta})^2} < 0,
\]
and
\[
\frac{\partial Ep^*(k)}{\partial \gamma} = \left( \rho + (1 - \rho) \frac{\gamma (1 - \delta) \frac{T}{\delta}}{1 + (1 - \delta) \frac{T}{\delta}} \right) \frac{\partial D_2(k, 1)}{\partial \gamma} - (1 - \rho) \frac{(V(k) - \bar{\tau} - D_2(k, 1)) (1 - \delta) \frac{T}{\delta} + V(k) - (1 - \delta) \frac{T}{\delta}}{1 + (1 - \delta) \frac{T}{\delta}} < 0,
\]
which completes the proof. ■

**Proof of Proposition 5:** When \( \rho < \rho^* \), the first order condition for \( k^* \) is given by
\[
\frac{dY^{NI}(k)}{dk} = \frac{\partial D_2(k, 0)}{\partial k} - 1 = \left( \frac{1 + (1 - \delta) \frac{T}{\delta}}{1 + (1 - \delta) (1 - \gamma) \frac{T}{\delta}} \right) (1 - \gamma) V'(k) - 1 \tag{22}
\]
and
\[
(1 - \gamma (1 - \rho *)) V'(k) - 1 = 0.
\]

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where the last equality follows by using (10). Since \( V''(k) < 0 \), the first order condition is sufficient for a maximum.

As mentioned in the text, when \( \rho > \rho^* \), the firm issues debt with face value \( D_2(k, 1) \).
With probability \( \rho \), the regulator is committed and sets a price \( p^* (D_2(k, 1), k, 1) = D_2(k, 1) + \bar{c} \), which ensures that the firm is immune to financial distress. With probability \( 1 - \rho \), the regulator is opportunistic and sets a price \( p^* (D_2(k, 1), k, 0) = D_1(k, 0) + \bar{c} + M (D_2(k, 1), 0) \) which leaves the firm susceptible to financial distress with probability \( \phi^T (k) \). Substituting \( p^* (D_2(k, 1), k, 1) \) and \( p^* (D_2(k, 1), k, 0) \) in equation (9), using equation (12), and rearranging terms, yields

\[
Y^T (k) \equiv Y (D_2(k, 1), k) = \rho \left( \frac{p^* (D_2(k, 1), k, 1)}{D_2(k, 1) + \bar{c}} \right) + (1 - \rho) \left[ \frac{p^* (D_2(k, 1), k, 0)}{D_1(k, 0) + \bar{c} + M (D_2(k, 1), 0)} \right]
- (1 - \rho)(1 - \delta) \phi^T (k) T - \left( 1 - \delta \right) \frac{\bar{c}}{2} - k
= \left( \rho - (1 - \gamma)(1 - \rho)(1 - \delta) \frac{T}{\bar{c}} \right) D_2(k, 1)
+ (1 - \rho)\left( 1 + (1 - \delta) \frac{T}{\bar{c}} \right) D_1(k, 0) + (1 + \delta) \left( 1 + \gamma (1 - \rho)(1 - \delta) \frac{T}{\bar{c}} \right) \frac{\bar{c}}{2} - k.
\]

Using the definitions of \( D_1(k, 0) \) and \( D_2(k, 1) \) and equation (10), yields equation (14) in the text. Differentiating this equation, yields the first order condition for \( k^* \):

\[
\frac{dY^T (k)}{dk} = (1 - \gamma(1 - \rho^*)) V''(k) - (1 - \gamma(\rho - \rho^*)) = 0. \tag{23}
\]

Since \( V''(k) < 0 \), the first order condition is sufficient for a maximum.

Equation (22) shows that \( k^* \) is independent of \( \rho \) when \( \rho^* < \rho \). Fully differentiating equation (23) with respect to \( k \) and \( \rho \) shows that when \( \rho > \rho^* \),

\[
\frac{\partial k^*}{\partial \rho} = -\frac{\gamma}{(1 - \gamma(1 - \rho^*)) V''(k)} > 0,
\]

where the inequality follows because \( V(\cdot) \) is concave, so \( V''(k) < 0 \). \( \blacksquare \)

**Proof of Proposition 6:** First, note from (10) that

\[
\frac{\partial \rho^*}{\partial \delta} = -(1 - \gamma)(1 - \rho^*)^2 \frac{T}{\bar{c}} < 0, \quad \frac{\partial \rho^*}{\partial \gamma} = -\frac{\rho^*(1 - \rho^*)}{1 - \gamma} < 0. \tag{24}
\]
When $\rho < \rho^*$, $k^*$ is implicitly defined by equation (22). Totally differentiating this equation with respect to $k$ and $\delta$, and recalling that $V''(\cdot) < 0 < V'(\cdot)$, yields

$$\frac{\partial k^*}{\partial \delta} = -\frac{\gamma \frac{\partial \rho^*}{\partial \delta} V'(k^*)}{(1 - \gamma (1 - \rho^*)) V''(k^*)} < 0.$$  

(25)

Similarly, totally differentiating equation (22) with respect to $k$ and $\gamma$,

$$\frac{\partial k^*}{\partial \gamma} = -\frac{\left(\frac{\partial \rho^*}{\partial \gamma} - (1 - \rho^*)\right) V'(k^*)}{(1 - \gamma (1 - \rho^*)) V''(k^*)} < 0.$$  

(26)

Next, suppose that $\rho > \rho^*$. Then $k^*$ is defined by (23). Totally differentiating this equation and noting from (33) that $V'(k^*) > 1$,

$$\frac{\partial k^*}{\partial \delta} = -\frac{\gamma \frac{\partial \rho^*}{\partial \delta} (V'(k^*) - 1)}{(1 - \gamma (1 - \rho^*)) V''(k^*)} < 0,$$

(27)

and

$$\frac{\partial k^*}{\partial \gamma} = -\frac{(1 - \rho^*) V'(k^*) + (\rho - \rho^*) + \gamma \frac{\partial \rho^*}{\partial \gamma} (V'(k^*) - 1)}{(1 - \gamma (1 - \rho^*)) V''(k^*)} < 0.$$  

(28)

Finally, to examine the effect of $\rho$ on $\frac{\partial k^*}{\partial \delta}$ and $\frac{\partial k^*}{\partial \gamma}$, we need to compare equation (25) with equation (27) and equation (26) with equation (28). To this end, let $k^{NI}$ and $k^I$ be the investment levels determined by (22) and (23). Then,

$$-\frac{\gamma \frac{\partial \rho^*}{\partial \delta} (V'(k^I) - 1)}{(1 - \gamma (1 - \rho^*)) V''(k^I)} > -\frac{\gamma \frac{\partial \rho^*}{\partial \delta} V'(k^I)}{(1 - \gamma (1 - \rho^*)) V''(k^I)},$$  

R.H.S. of equation (27)

$$> -\frac{\gamma \frac{\partial \rho^*}{\partial \delta} V'(k^{NI})}{(1 - \gamma (1 - \rho^*)) V''(k^{NI})},$$  

R.H.S. of equation (25)

where the first inequality follows since $\frac{\partial \rho^*}{\partial \delta} < 0$, and the second follows since $\frac{V'(k)}{V''(k)}$ is nondecreasing and since Proposition 5 implies that $k^I > k^{NI}$. Similarly,

$$-\frac{\left(\frac{\partial \rho^*}{\partial \gamma} - (1 - \rho^*)\right) (V'(k^I) - 1) - (1 - \rho)}{(1 - \gamma (1 - \rho^*)) V''(k^I)} > -\frac{\left(\frac{\partial \rho^*}{\partial \gamma} - (1 - \rho^*)\right) V'(k^I)}{(1 - \gamma (1 - \rho^*)) V''(k^I)}$$  

R.H.S. of equation (28)

$$> -\frac{\left(\frac{\partial \rho^*}{\partial \gamma} - (1 - \rho^*)\right) V'(k^{NI})}{(1 - \gamma (1 - \rho^*)) V''(k^{NI})},$$  

R.H.S. of equation (26)
where the first inequality follows since $\rho > \rho^*$ when the regulator is independent and since 
$\frac{\partial \rho^*}{\partial \gamma} < 0$, and the second inequality follows since $\frac{V'(k)}{V''(k)}$ is nondecreasing and since $k^I > k^{NI}$.

Proof of Proposition 7: In equilibrium, $D = D_2(k^*, 0)$ if $\rho < \rho^*$ and $D = D_2(k^*, 1)$ if $\rho > \rho^*$. Equation (8) shows that $D_2(k^*, I)$ is affected by $\rho$ only through the choice of $k$ but not directly. Using equations (6) and (8) and the definition of $\rho^*$ in Proposition 1,

$$\frac{dD_2(k^*, I)}{dk} = \left( \frac{1 + (1 - \delta) \frac{T}{\tau}}{1 + (1 - \gamma) (1 - \delta) \frac{T}{\tau}} \right) \frac{dD_1(k^*, I)}{dk} - \frac{\gamma I (1 - \delta) \frac{T}{\tau}}{1 + (1 - \gamma) (1 - \delta) \frac{T}{\tau}}$$

$$= \left( \frac{1 + (1 - \delta) \frac{T}{\tau}}{1 + (1 - \gamma) (1 - \delta) \frac{T}{\tau}} \right) ((1 - \gamma) V'(k^*) + \gamma I)$$

$$= \frac{\gamma I (1 - \delta) \frac{T}{\tau}}{1 + (1 - \gamma) (1 - \delta) \frac{T}{\tau}}$$

$$= (1 - \gamma (1 - \rho^*)) V'(k^*) + \gamma I (1 - \rho^*) > 0$$

Hence, both $D_2(k^*, 0)$ and $D_2(k^*, 1)$ are increasing with $k$. As in the proof of Proposition 6, let $k^{NI}$ and $k^I$ denote the equilibrium levels of investment when the regulator is non independent ($\rho < \rho^*$) and when he is independent ($\rho > \rho^*$) and recall that $k^I > k^{NI}$ by Proposition 4. Then,

$$D_2(k^{NI}, 0) < D_2(k^I, 0) < D_2(k^I, 1),$$

where the second inequality follows because if we hold $k$ fixed, $D_2(k, 0) < D_2(k, 1)$.

Next, we consider the effects of $\delta$ and $\gamma$ on the firm’s debt. Proposition 2 shows that holding $k$ fixed, $\delta$ and $\gamma$ have a negative direct effect on debt. Equation (29), together with Proposition 6, implies that the indirect effect is negative as well. Hence, the equilibrium level of debt is decreasing with $\delta$ and $\gamma$, even after the endogenous choice of $k$ is taken into account.

As for the regulated price, recall from Corollary 1 that it is given by $D_2(k^*, 0) + \bar{c}$ if $\rho < \rho^*$ and by $Ep^*(D_2(k^*, 1), k^*)$ if $\rho > \rho^*$. Given that $k^*$ is independent of $\rho$ when $\rho < \rho^*$, but is increasing with $\rho$ when $\rho > \rho^*$, it follows that

$$D_2(k^{NI}, 0) + \bar{c} < D_2(k^I, 0) + \bar{c} < Ep^* \left( D_2 \left( k^I, 1 \right), k^I \right),$$

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where the right inequality follows by Proposition 3 which states that if we hold $k$ fixed, the expected price is higher when the regulator is independent. Therefore, the regulated price is higher when $\rho > \rho^*$ than when $\rho < \rho^*$.

Since $D_2(k^*, 0)$ is decreasing with $\delta$ and $\gamma$, the regulated price is also decreasing with $\delta$ and $\gamma$ for all $\rho < \rho^*$. When $\rho > \rho^*$, equation (11) implies that

$$
\frac{dE_p(k^*)}{dk} = \rho \frac{dD_2(k^*, 1)}{dk} + (1 - \rho) \left( \frac{dD_1(k^*, 0)}{dk} + \frac{\partial M(D_2(k, 1), 0) dD_2(k^*, 1)}{\partial D} \right) > 0, \tag{30}
$$

where the inequality follows by (29) and since $\frac{dD_1(k, 0)}{dk} = (1 - \gamma) V'(k) > 0$ and since $\frac{\partial M(D_2(k, 1), 0)}{\partial D} > 0$ by equation (7). Together with Proposition 6, it follows that $\delta$ and $\gamma$ have a negative indirect effect on $E_p(k^*)$. Proposition 2 in turn shows that holding $k$ fixed, the direct effect is also negative. Hence, the regulated price is decreasing with $\delta$ and $\gamma$ when $\rho > \rho^*$.

Finally, recall that when $\rho > \rho^*$, the probability of financial distress is $\phi^I(k^*)$, where $\phi^I(k)$ is given by (12). Since $\frac{\partial k^*}{\partial \rho} > 0$ by Proposition 5, $\phi^I(k^*)$ is increasing with $\rho$.

Using (27), (24), and (10),

$$
\frac{d\phi^I(k^*)}{d\delta} = \frac{\gamma}{\overline{c}} \left( \frac{\partial k^*}{\partial \delta} \frac{T}{\bar{\sigma}} \right) + \frac{\gamma k^* T}{\overline{c}^2 (1 + (1 - \delta) \frac{T}{\bar{\sigma}})^2} \left[ \frac{\gamma (1 - \gamma) (1 - \rho^*)^2 (V'(k^*) - 1)}{(1 - \gamma (1 - \rho^*)) V''(k^*)} + \frac{k^* (1 - \gamma (1 - \rho^*))}{1 + (1 - \delta) \frac{T}{\bar{\sigma}}} \right]
$$

$$
= \frac{\gamma}{\overline{c}^2 (1 + (1 - \delta) \frac{T}{\bar{\sigma}})^2} \left[ \frac{\gamma (1 - \gamma) (1 - \rho^*)^2 (V'(k^*) - 1) T k^*}{(1 - \gamma (1 - \rho^*)) V''(k^*)} \right]
$$

$$
= \frac{\gamma}{\overline{c}^2 (1 + (1 - \delta) \frac{T}{\bar{\sigma}})^2} \left[ \frac{\gamma (1 - \gamma) (1 - \rho^*)^2 T k^*}{(1 - \gamma (1 - \rho^*)) V''(k^*)} \right]
$$

$$
> \frac{\gamma}{\overline{c}^2 (1 + (1 - \delta) \frac{T}{\bar{\sigma}})^2} \left[ \frac{\gamma (1 - \gamma) (1 - \rho^*)^2 T k^*}{(1 - \gamma (1 - \rho^*)) V''(k^*)} \right].
$$

The condition in the proposition ensures that the square bracketed term, and hence the entire derivative, are positive.
Likewise, using (28), (24), and (10),
\[
\frac{d\phi^I(k^*)}{d\gamma} = \frac{\gamma \frac{\partial k^*}{\partial \gamma} + k^*}{c\left(1 + (1 - \delta) \frac{T}{\gamma}\right)}
\]
\[
= \frac{k^*}{c\left(1 + (1 - \delta) \frac{T}{\gamma}\right)} \left[ \frac{1 - \rho^*}{1 - \gamma} \frac{V'(k^*)}{V''(k^*)k^*} - 1 + \frac{1 - \rho^*}{1 - \gamma} + 1 \right]
\]
\[
> \frac{k^*}{c\left(1 + (1 - \delta) \frac{T}{\gamma}\right)} \left[ \frac{\gamma (1 - \rho^*)}{1 - \gamma} \frac{V'(k^*)}{V''(k^*)k^*} + 1 \right]
\]
\[
= \frac{k^* \gamma (1 - \rho^*)}{c\left(1 + (1 - \delta) \frac{T}{\gamma}\right)} \left[ \frac{V'(k^*)}{V''(k^*)k^*} + \frac{1 - \gamma}{\gamma (1 - \rho^*)} \right]
\]
\[
= \frac{k^* \gamma (1 - \rho^*)}{c\left(1 + (1 - \delta) \frac{T}{\gamma}\right)} \left[ \frac{V'(k^*)}{V''(k^*)k^*} + \frac{(1 - \gamma)(1 + (1 - \gamma)(1 - \delta) \frac{T}{\gamma})}{\gamma} \right],
\]
where the first inequality follows because \( V''(k^*) < 0 \) and \( \rho > \rho^* \) imply that \( \frac{1 - \rho^*}{V''(k^*)k^*} > \frac{1 - \rho}{V''(k^*)k^*} \). The condition in the proposition ensures that the square bracketed term, and hence the entire derivative, are positive.  

Proof of Proposition 8: We first compare the equilibrium level of investment, \( k^* \), with the socially optimal level. To this end, note that when \( \rho < \rho^* \), the first best level of investment maximizes \( W^{NI}(k) \) and hence is implicitly defined by the first order condition \( V'(k) = 1 \). Since equation (22) implies that \( k^* \) is such that
\[
V'(k^*) = \frac{1}{1 - \gamma (1 - \rho^*)} > 1,
\]
the firm underinvests relative to the first best.

When \( \rho > \rho^* \), the first best level of investment maximizes \( W^I(k) \). Now, the first order condition for the first best level of investment is
\[
V'(k) = 1 + \frac{\gamma (1 - \rho)}{1 + (1 - \delta) \frac{T}{\gamma}} = \frac{1 + (1 - \delta) \frac{T}{\gamma} + \gamma (1 - \rho) \frac{T}{\gamma}}{1 + (1 - \delta) \frac{T}{\gamma}}.
\]
On the other hand, equation (23) implies that \( k^* \) is such that
\[
V'(k^*) = \frac{1 - \gamma (\rho - \rho^*)}{1 - \gamma (1 - \rho^*)} > 1.
\]
Now notice that the right-hand side of (33) exceeds the right-hand side of (32):
\[
\frac{1 - \gamma (\rho - \rho^*)}{1 - \gamma (1 - \rho^*)} - \frac{1 + (1 - \delta) \frac{T}{\bar{e}} + \gamma (1 - \rho) \frac{T}{\bar{e}}}{1 + (1 - \delta) \frac{T}{\bar{e}}} = \frac{\gamma (1 - \rho) \left(1 - (1 - \gamma) (1 - \rho^*) \frac{T}{\bar{e}}\right)}{1 - \gamma (1 - \rho^*)} > 0.
\]
Since $V'(k)$ is decreasing, $k^*$ is lower than the first best level of investment.

Next, we turn to the comparative statics of welfare. When $\rho < \rho^*$, the equilibrium value of welfare is given by $W^{NI}(k^*)$. Differentiating with respect to $x = \rho, \delta, \gamma$, yields
\[
\frac{\partial W^{NI}(k^*)}{\partial x} = [V'(k^*) - 1] \frac{dk^*}{dx}.
\]
Since equation (31) implies that $V'(k^*) > 1$, and since Propositions 5-6 imply that when $\rho < \rho^*$, $\frac{dk^*}{d\rho} = 0$, $\frac{dk^*}{d\delta} < 0$, and $\frac{dk^*}{d\gamma} < 0$, we get $\frac{\partial W^{NI}(k^*)}{\partial \rho} = 0$, $\frac{\partial W^{NI}(k^*)}{\partial \delta} < 0$, and $\frac{\partial W^{NI}(k^*)}{\partial \gamma} < 0$.

When $\rho > \rho^*$, the equilibrium value of welfare is given by $W^I(k^*)$. Differentiating with respect to $\rho$, yields
\[
\frac{\partial W^I(k^*)}{\partial \rho} = \left[V'(k^*) - 1 - \frac{(1 - \rho) \gamma \frac{T}{\bar{e}}}{1 + (1 - \delta) \frac{T}{\bar{e}}} \right] \frac{dk^*}{d\rho} + \frac{\gamma k^* \frac{T}{\bar{e}}}{1 + (1 - \delta) \frac{T}{\bar{e}}},
\]
where the second equality follows by substituting for $V'(k^*)$ from (33) and the third equality follows by substituting for $\rho^*$ from (10) and simplifying. By Proposition 5, $\frac{dk^*}{d\rho} > 0$. Hence, $1 - \delta (1 - \gamma) \frac{T}{\bar{e}} > 0$ is sufficient for $\frac{\partial W^I(k^*)}{\partial \rho} > 0$.

Likewise, differentiating $W^I(k^*)$ with respect to $\delta$ and $\gamma$, using (33) and (10) and simplifying, yields
\[
\frac{\partial W^I(k^*)}{\partial \delta} = \frac{(1 - \rho) \gamma}{(1 - \gamma) \left(1 + (1 - \delta) \frac{T}{\bar{e}}\right)} \left[1 - \delta (1 - \gamma) \frac{T}{\bar{e}}\right] \frac{dk^*}{d\delta} - \frac{(1 - \rho) \gamma k^* \left(\frac{T}{\bar{e}}\right)^2}{\left(1 + (1 - \delta) \frac{T}{\bar{e}}\right)^2},
\]
and
\[
\frac{\partial W^I(k^*)}{\partial \gamma} = \frac{(1 - \rho) \gamma}{(1 - \gamma) \left(1 + (1 - \delta) \frac{T}{\bar{e}}\right)} \left[1 - \delta (1 - \gamma) \frac{T}{\bar{e}}\right] \frac{dk^*}{d\gamma} - \frac{(1 - \rho) \gamma k \left(\frac{T}{\bar{e}}\right)^2}{\left(1 + (1 - \delta) \frac{T}{\bar{e}}\right)^2}.
\]
Recalling from Proposition 6 that $\frac{dk^*}{d\delta} < 0$ and $\frac{dk^*}{d\gamma} < 0$, it follows that $1 - \delta (1 - \gamma) \frac{T}{\bar{e}} > 0$ is sufficient for $\frac{\partial W^I(k^*)}{\partial \delta} < 0$ and $\frac{\partial W^I(k^*)}{\partial \gamma} < 0$. 

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9 References


