Network competition with income effects*

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February 2, 2011

Abstract

I generalize the workhorse model of network competition to include income effects in call demand. Empirical work has shown call demand to increase significantly with income. Weak income effects are enough to deliver results in line with empirical regularities when networks are differentiated: the networks have an incentive to agree on high termination rates to soften competition. This holds with or without call price discrimination. Under price discrimination the networks charge a lower price for calls within the own network (on-net) than to other networks (off-net).

Keywords: income effects, network competition, termination-based price discrimination, two–part tariffs, profit neutrality, termination rates.
JEL classification: L51, L96

*Many thanks to Johan Stennek, Joacim Tåg and seminar participants at IFN for their valuable comments and to Max Brimberg for research assistance. Financial support from Almega as well as the NET Institute (www.netinst.org) is gratefully acknowledged.

1 Introduction

The seminal contributions on network competition (Armstrong, 1998; Laffont, Rey and Tirole, 1998a,b) generate a number of puzzling predictions concerning the termination rates the networks charge for connecting calls from one another and regarding call prices. First, theory predicts network profit to be independent of the termination rate when the networks compete in two-part call tariffs and charge the same price for all calls (Laffont, Rey and Tirole, 1998a). In reality, profit margins tend to fall whenever networks are forced by regulators to lower their termination rates (Genakos and Valletti, 2009). This discrepancy between predicted and observed effects on profit constitutes a *profit neutrality puzzle*.[1] Profit is no longer independent of the termination rate when the networks engage in price discrimination between calls inside the network (on-net) and calls to other networks (off-net); see Laffont, Rey and Tirole (1998b). Yet, price discrimination addresses the profit neutrality puzzle only to introduce another. The networks now have an incentive to agree on termination rates *below* cost (Gans and King, 2001). As off-net calls then have a lower perceived marginal cost than on-net calls, the workhorse model predicts off-net call prices below on-net call prices. In reality, off-net calls are nearly always more expensive than on-net calls under price discrimination. The discrepancy between predicted and observed call prices constitutes an *off-net price puzzle*.

In this paper I show that introducing income effects in call demand can solve both the profit neutrality puzzle and the off-net price puzzle: in the standard case of differentiated networks, the networks have an incentive to agree on termination rates above cost already for weak income effects. This holds independently of whether the networks engage in call price discrimination or not. A markup on termination implies that off-net calls are more expensive than on-net calls under price discrimination.

Extending the model to include income effects is empirically relevant. With income effects, call demand generally depends on the price of all types of calls, the subscription fee and on income. In a study of residential telephony in France, Aldebert et al. (2004) find consumers in higher income classes to display significantly higher demand for local and national calls than consumers in lower income classes, and there are significant cross-price elasticities between local, national and international calls.[2]

[1] Under profit neutrality the networks should not oppose to lowering the termination rates at the regulator's request. This is *not* how networks normally respond to tighter regulation. Sweden constitutes an illustrative case in point here. In 2004, the Swedish regulatory agency for telecommunications, PTS, deemed all four mobile networks, TeliaSonera, Tele2, Vodafone (later Telenor) and Hi3G to have significant market power and instructed them to lower their termination rates. Every summer since then PTS has required the operators to further reduce their rates. Over the three years following regulation the operators consistently refused to lower termination rates. The only exception was TeliaSonera who voluntarily lowered the rate on one occasion, in 2007. The termination rate disputes have been settled in court from 2008 and onwards. The final rulings have been in favour of PTS on every account. Apparently, the operators have given up the fight. Since 2008, the operators have only sporadically refused to lower their termination rates in accordance with PTS' demands.

[2] Acton and Vogelsang (1992) and Munoz and Amaral (1996) present additional evidence of income effects by reporting a significant relationship between demand for international calls and aggregate measures of income such as GDP and Gross Value Added - see also the references therein. The cited studies are all on fixed-line call demand. I have found no references to empirical analyses of call demand in mobile telephony. However, there is no obvious reason why mobile call demand should be completely independent of income while fixed telephony call demand in some cases is highly dependent on income.

Considering income effects is policy relevant. Based upon the results of the workhorse model one would conclude that the conditions for efficient regulation were favourable. Policy makers could implement the first-best welfare optimum by means of a simple cost-based regulation: disallow call price discrimination and demand termination rates equal to reported marginal termination cost. As the networks do not care about the termination rate under uniform pricing, they have no incentive to lie about marginal cost. This paper shows that regulated networks instead have a strong incentive to exaggerate marginal costs under cost-based regulation, even for weak income effects. A well designed regulatory policy needs to take these incentives into account instead of just accepting reported costs at face value.

In the workhorse model, the networks maximize network surplus by charging call prices equal to perceived marginal cost and instead use the subscription fee to compete for customers. Thus, network profit stems entirely from the subscription fee and the termination profit on incoming calls. Whereas a high termination rate is a means to increasing termination profit, it simultaneously drives down the equilibrium subscription fee. Under uniform call prices (i.e. absent price discrimination), the network can save on termination payments by completing a larger share of initiated calls inside the own network. This cost-saving incentive renders it more profitable to slash the subscription fee and attract more customers the higher is the termination rate. With total market size being constant, the networks' incentive to save on call costs only intensifies competition for subscribers. The marginal negative effect on the subscription fee exactly offsets the marginal increase in termination profit, leaving total profit independent of the termination rate under uniform prices. When all calls cost the same, consumers do not care about the size of the network they belong to. Size becomes important for the choice of network whenever networks price discriminate between calls inside the own network and calls to other networks. If on-net calls are cheaper that off-net calls, as is usually the case, consumers are drawn to the largest network to save on call expenditures. The larger the network, the more advantageous it is to belong to it. Cutting the subscription fee becomes extra profitable to the individual network in this case of tariff-mediated network externalities (Laffont, Rey and Tirole, 1998b) because increased network size attracts additional customers. With a constant market size, this network multiplier only serves to reinforce competition for subscribers and drive down equilibrium subscription fees. The networks can soften competition by lowering termination rates because the network multiplier is weaker the cheaper are off-net calls in relation to on-net calls. As Gans and King (2001) show, the networks jointly prefer a termination rate below cost. When termination is priced below cost, the off-net price is lower than the on-net price, and consumers prefer the *smallest* network.

Introducing income effects opens a new channel through which high termination rates soften competition for subscribers. Subscription demand depends on the consumer net surplus each network offers its customers. Consumer net surplus includes call utility less call expenditures and the cost of the subscription. The lower is the marginal utility of income, the less important is the subscription fee for the choice of network. Marginal utility of income is lower the less the consumers spend on calls because of a higher residual income. Call expenditures are lower the more expensive are calls. High call prices are exactly what the networks achieve by agreeing on high termination rates. Thus, a higher termination rate means a lower marginal utility of income, a lower subscription elasticity and by implication softer competition.

Even an infinitesimal income effect softens competition enough to tilt networks to favour very high termination rates absent call price discrimination. This is because profit neutrality is a knife-edge result: the marginal effect on the subscription fee is exactly proportional to the marginal effect on termination profit. Under call price discrimination, a stronger income effect is generally needed to generate an optimal termination rate above cost because the networks strictly prefer a termination rate below cost absent any income effects. If networks are horizontally differentiated, the network multiplier is small in magnitude compared to marginal utility of income. In this case, a weak income effect is enough to render a termination rate above cost profitable even under call price discrimination.[3] To summarize; income effects in call demand can solve both the profit neutrality puzzle and the off-net price puzzle.

The puzzling predictions of the basic model have stimulated extensions of the workhorse model in many directions. Despite the numerous articles on network competition, only two offer solutions to both the profit neutrality puzzle and the off-net price puzzle - I summarize recent papers and main contributions in Section 2. Armstrong and Wright (2009) analyze network competition when a monopoly fixed-line operator demands call termination from competing suppliers of mobile call termination. The mobile operators would like to set low termination rates between themselves to soften competition for subscribers, but charge a high termination rate from the fixed-line operator to exercise vertical market power. If the fixed-line operator can bypass termination by transiting calls via the competitor's mobile network, it becomes less profitable to uphold large differences between fixed and mobile termination rates. For significant arbitrage possibilities and under the additional assumption that fixed-line termination profit is sufficiently important to mobile operators, all call termination is priced above cost.[4] For significant arbitrage possibilities and under the additional assumption that fixed-line termination profit is sufficiently important to mobile operators, all call termination is priced above cost. Jullien et al. (2010) study network competition when consumers are heterogenous and total market size is price elastic. A proportion of subscribers are "light" users, and the rest are "heavy" users. Light users hold subscriptions only because they value receiving calls, they do not initiate calls. The presence of light users softens competition for heavy users because a heavy user in the competitor's network generates a larger termination profit than a heavy user in the own network. Jullien et al. (2010) show that the networks then agree on termination rates above cost.

The present paper is the first to analyze the consequences of income effects in call demand on network competition and to show that the networks may then have an incentive to charge a markup on termination. Thus, I generate a new empirical prediction: a downward regulation of termination rates should have a negative impact on profit margins of mobile networks even

[3] Assuming a high degree of network differentiation is very common in the literature on network competition. Network differentiation allows a high degree of freedom in the choice of termination rates, while preserving concavity of the profit function and uniqueness of subscription demand.

[4] Actually, Armstrong and Wright (2009) assume call price discrimination throughout their analysis. However, their main result on termination rates trivially extends to the case of uniform pricing; see Section 2.

after controlling for total subscription elasticity and arbitrage opportunities.

2 Literature

This Section summarizes the most recent contributions and a number of key papers on optimal termination rates under network competition. Most papers in the literature fall in one (or more) of the four specific categories in Table 1 below.[5]

	PN puzzle	OFF puzzle
Consumer expectations		
Hurkens and López (2010a)		+
Hoernig et al. (2009)		+
Gabrielsen and Vagstad (2008)		+
Heterogenous consumers		
Jullien et al. (2010)	+	+
Dessein (2003 and 2004)	- and +	
Hahn (2004)	-	
Call externalities		
Hurkens and López (2010b)	-	-
Berger (2005)		-
Jeon et al. (2004)	-	
Armstrong (2002)	-	
Entry		
López and Rey (2009)	-	+
Calzada and Valletti (2008)		+
Other		
Armstrong and Wright (2009)	(+)	+
Valletti and Cambini (2005)	+	
The present study	+	+

Table 1: Summary of the network competition literature

A "+" in Table 1 indicates that the paper in question solves the relevant puzzle, i.e. predicts a strictly positive markup on termination under uniform call prices (the PN puzzle) respective an off-net price above the on-net price under price discrimination (the OFF puzzle). A "-" shows that the paper does address the puzzle, but fails to solve it, whereas a blank space indicates that the paper does not address the relevant puzzle.

In the workhorse model with price discrimination consumers take into account the effect a price reduction has on network size when they choose which network to subscribe to. This network multiplier reinforces competition. Hurkens and López (2010a) explore the consequences of assuming instead that consumers do not take into account the effect of a price increase on

[5] An extensive review is outside the scope of this paper, but I refer the interested reader to the most recent papers in each of the categories in Table 1 for a more complete list of references. See also Armstrong (2002) and Vogelsang (2003) for comprehensive surveys of the earlier literature.

network size, they have *passive expectations* about network size. The network multiplier vanishes under passive expectations, so the termination rate is set only to maximize termination profit.[6] Fully responsive consumers (as in the workhorse model) and completely passive consumers (as in Hurkens and López, 2010a) represent two extreme representations of consumer expectations. An intermediate stand is to assume that every subscriber only takes the actions of some other customers into account - consumers belong to so called "calling clubs" (Gabrielsen and Vagstad, 2008; Hoernig et al., 2009). The smaller is the calling club, the weaker is the network multiplier and the higher is the termination rate.[7] Analyzing consumer expectations certainly leads to a deeper understanding of network competition, yet the policy implications are less obvious. The regulator could still implement the first-best policy by requiring uniform pricing and cost-based termination rates: consumer expectations about network size do not matter under uniform pricing.

All consumers have identical calling patterns in the workhorse model. Jullien et al. (2010) assume instead that a proportion of subscribers are "light" users, and the rest are "heavy" users. Light users hold subscriptions only because they value receiving calls, they do not initiate calls. Aggregate subscription demand of light users is price elastic, whereas aggregate demand of heavy users (the ones who initiate calls) is constant. The presence of light users softens competition for heavy users because a heavy user in the competitor's network generates a larger termination profit than a heavy user in the own network. Jullien et al. (2010) show that the networks then agree on termination rates above cost. The assumption of elastic total subscription demand appears crucial, and the result is sensitive to the exact specification of total subscription demand. Dessein (2003) and Hahn (2004) show that profit neutrality holds in the workhorse model also under consumer heterogeneity, assuming constant aggregate demand of heavy and light users. Dessein (2003) as well as Armstrong and Wright (2009) consider elastic aggregate subscription demand. The first paper assumes a "logistic model" of subscription demand, whereas the second paper assumes a "Hotelling model with hinterlands". In both papers, it is more profitable to lower the termination rate marginally below marginal cost than to increase it marginally. Probably, the assumption that demand of light users is more elastic than demand of heavy users is important. In a model with constant market size, Dessein (2004) finds the profit maximizing termination rate to be below (above) marginal cost if light user demand is less (more) elastic than heavy user demand. To summarize, elastic total demand generates ambiguous results concerning the incentives for distorting the termination rate. Be as it may, the workhorse model's assumption of a fully covered market is an increasingly fitting description of more and more national markets. In 2007, 22 OECD countries had more than 100 mobile subscribers per 100 inhabitants (OECD, 2009). The future prospects for efficient cost-based regulation

[6] Passive expectations are related to the notion of competing in utilities instead of prices (see e.g. Calzada and Valletti, 2008). Networks compete in utilities by guaranteeing their subscribers a certain surplus independently of the size of the network. Then, network size does not matter to consumers even in the case of call price discrimination. Passive expectations and competition in utilities are not equivalent, however. In the latter case, the network has to adjust the pricing plan to account for changes in market share and keep surplus constant.

[7] This result rests on the assumption that the members of the calling club do *not* coordinate the choice of network. In case of coordination, calling clubs have no effect on the optimal termination rate (Calzada and Valletti, 2008; Gabrielsen and Vagstad, 2008).

of termination rates under uniform pricing thus appear bright in many countries, even with heterogenous consumers (Dessein, 2003; Hahn, 2004).

The workhorse model rests on the assumption that consumers only care about initiated calls. Naturally, consumers may derive utility also from receiving calls. Call externalities have no bearing on competition under uniform pricing. Whereas call externalities do affect the value of holding a subscription, the magnitude of the externality is independent of the network one subscribes to and therefore unimportant for the choice of network. Under uniform pricing, network profit is independent of the termination rate whether the networks charge for incoming calls (Jeon et al., 2004; Hurkens and López, 2010b) or not (Armstrong, 2002). Under call price discrimination, the optimal termination rate lies below cost when the networks charge for incoming calls (Hurkens and López, 2010b) and when they do not (Berger, 2005). Note also that call externalities may have dramatic effects on competition that do not work through the termination rate: with strong call externalities and network externalities, the networks may prefer not to connect at all (Jeon et al., 2004; Berger, 2005; Hurkens and López, 2010b).

Calzada and Valletti (2008) as well as López and Rey (2009) consider the possibility of using reciprocal termination rates to deter entry. In Calzada and Valletti's (2008) model, profit is decreasing in the termination rate just as in the workhorse model with price discrimination. The incumbent operators may nevertheless prefer lower network profit if those lower profits guarantee that potential entrants stay out of the market because they cannot cover their entry costs. Entry deterrence by means of an excessive termination rate here relies on the assumption that the incumbents can commit to an industry-wide termination rate (i.e. one that applies even to all potential entrants). If the termination rate is renegotiated posterior to entry, the profit maximizing termination rate lies below cost. López and Rey (2009) claim that consumer inertia and a high reciprocal termination rate can help the incumbent sustain monopoly. Inertia and network externalities render it too costly for an entrant to build market share. A prerequisite for entry deterrence is price discrimination and resulting network externalities. Under uniform pricing, the incumbent either cannot distort termination rates to foreclose entry or has no incentive to do so (Carter and Wright, 2003; López and Rey, 2009). Consequently, cost-based regulation of termination rates and a requirement that networks charge a uniform price for all calls guarantee marginal cost pricing of termination and free entry.

Valletti and Cambini (2005) assume that networks compete in quality as well as prices. Investing in quality is pure waste from the viewpoint of the industry: the size of the market is constant, and quality only intensifies competition for subscribers. The networks therefore prefer to minimize quality to save on cost. A marginal increase in the termination rate above cost is shown to reduce investment in quality, which leads Valletti and Cambini (2005) to the conclusion that the networks prefer termination rates above cost to curb investments. However, they do not show that an excessive termination rate is better than *all* termination rates below cost.

Armstrong and Wright (2009) consider network competition when there is a fixed-line network with locked-in subscribers in addition to mobile operators competing for subscribers. As in the workhorse model, the mobile operators would like to set low termination rates between themselves to soften competition for subscribers, but charge a high termination rate from the fixed-line operator to exercise vertical market power. If the fixed-line operator can bypass termination by transiting calls via the competitor's mobile network, it becomes less profitable to uphold large differences between fixed and mobile termination rates. For significant arbitrage possibilities and under the additional assumption that fixed-line termination profit is sufficiently important to mobile operators, all call termination is priced above cost. Armstrong and Wright (2009) restrict attention to the case of price discrimination, but their result extends trivially to uniform prices. With uniform call prices, the networks do not care about mobile termination rates, but still care about fixed-line termination rates. Even the slightest arbitrage possibility would therefore cause the mobile termination rates to jump to the level of the fixed-line termination rates.[8] Based on the results of Armstrong and Wright (2009) and on market trends, one would predict falling termination rates even absent regulation. First, fixed-line termination is losing its importance as a source of profit for mobile operators. An extreme example is Finland, where close to 90% of all originating call minutes were mobile minutes in 2009, up from 30% in 2002. All Nordic countries display a similar trend (PTS, 2010). Second, the advent of broadband (VoIP) telephony has introduced network competition even in fixed-line telephony. In Norway and Denmark, nearly 30% of all fixed-line subscriptions were VoIP in 2009, up from less than 10% in 2005 (PTS, 2010).

The above papers all contribute to our understanding of the impact of termination rates on network competition by extending the workhorse model to account for relevant market characteristics. Despite the numerous articles on network competition, the papers by Armstrong and Wright (2009) and Jullien et al. (2010) appear to be the only ones besides the present one to offer an explanation to both the profit neutrality puzzle and the off-net price puzzle. Armstrong and Wright (2009) consider arbitrage possibilities in network termination, whereas Jullien et al. (2010) build on the assumption of elastic total call demand. I follow a different path and explore the consequences of income effects in call demand. Probably there is more than one reason for why the networks would want to agree on termination rates above cost. A main reason for considering income effects is their empirical relevance (Aldebert et al., 2004). Thus, excessive termination rates might continue to pose a problem for regulators even as countries approach full mobile penetration, more and more calls originate in the mobile networks and the fixed-line incumbent becomes increasingly exposed to network competition.

3 Uniform Call Prices: The Profit Neutrality Puzzle

The Model I generalize the workhorse model by Armstrong (1998) and Laffont, Rey and Tirole (1998a and b), henceforth A-*LRT*, to allow for income effects in call demand. A continuum of consumers with unit measure are uniformly distributed on the unit interval. Each consumer

[8] Armstrong and Wright's (2009) model relies on the assumption that the fixed-line termination rate is set non-cooperatively, simultaneously with retail prices and subsequent to the (reciprocal) mobile termination rate. Finding the equilibrium termination rates when fixed-line and mobile termination rates are set simultaneously and prior to retail prices, is still open for research.

subscribes to one of two networks located at each end of the interval. I assume in this section that all calls have the same price, whereas the next section allows networks to price discriminate between calls inside (on-net) and outside (off-net) one's own network. The call pattern is balanced: every subscriber to network $i = 1, 2$ places $q_i \geq 0$ calls to every other subscriber and consumes $y_i \geq 0$ of a numeraire good to maximize utility $U(q_i) + Z(y_i)$, subject to the budget constraint $p_i q_i + y_i + t_i \leq I$. The price per call is $p_i \geq 0$, the subscription fee equals t_i, and I is exogenous income. Call utility features constant elasticity, $U(q) = (1 - 1/\eta) q^{1-1/\eta}$, with $\eta > 1$. Consumption y of the numeraire good renders utility $Z(y) = y - \varepsilon y^2/2$, where $\varepsilon \geq 0$. The workhorse, A-LRT model, features quasi-linear utility: $\varepsilon = 0$.

Utility maximization yields call demand $D_i = D(p_i, t_i)$, demand $Y_i = Y(p_i, t_i)$ for the numeraire good and a shadow price of the budget constraint $\Lambda_i = \Lambda(p_i, t_i)$. A difference between this model and A-LRT is that call demand now decreases in the subscription fee t_i and not only in the call price p_i; see the Appendix for the details. Consumer net surplus in network i is

$$v_i = V(p_i, t_i) = U(D(p_i, t_i)) + Z(Y(p_i, t_i)) + \Lambda(p_i, t_i)(I - p_i D(p_i, t_i) - Y(p_i, t_i) - t_i). \quad (1)$$

The consumer located at $k \in [0, 1]$ derives utility $v_0 + v_1 - \tau k$ from subscribing to network 1 and utility $v_0 + v_2 - \tau(1 - k)$ of subscribing to network 2, where v_0 is the utility of holding a subscription, whereas τ is the virtual transportation cost and a measure of horizontal differentiation. The customer base of network i equals

$$S_i = \alpha(v_i, v_j) = \frac{1}{2} + \frac{v_i - v_j}{2\tau}, \; i \neq j = 1, 2, \quad (2)$$

when all consumers belong to one network or the other. The market is fully covered ($S_1 + S_2 = 1$) if the two networks offer similar tariffs ($v_i - v_j$ is small), or the networks are sufficiently differentiated (τ is large). I employ the standard assumption that τ is sufficiently high to render the market fully covered.

The profit of network i under uniform call prices equals

$$\Pi_i = S_i[\underbrace{S_i(p_i - c)D_i}_{\text{On-net call profit}} + \underbrace{(1 - S_i)(p_i - c_O - a)D_i}_{\text{Off-net call profit}} + \underbrace{t_i - f}_{\text{Subscription profit}} + \underbrace{(1 - S_i)(a - c_T)D_j}_{\text{Termination profit}}].$$

The term in brackets is the profit per subscriber. Each network derives its profits from four sources. The first term is the profit on initiated calls terminated inside the network (on-net), which is positive if the call price p_i exceeds the marginal call cost of on-net calls $c = c_O + c_T$, where c_O (c_T) is the marginal cost of call origination (termination). Second, the network profits from initiated calls terminated in the competitor's network (off-net) if the call price p_i exceeds the marginal cost of terminating a call off-net, which equals the marginal cost of call origination c_O plus the termination rate a. Third, the network earns a subscription profit if the subscription fee t_i is higher than the per-subscriber cost f. The final term constitutes the termination profit on incoming off-net calls, which is positive if the (reciprocal) termination rate a is higher than the marginal cost c_T of call termination.

Analysis Increasing the call price p_i leads to higher profit for a given customer base and a given number of initiated calls due to a higher markup. This is the first term in marginal profit below. However, the price increase comes at the cost of fewer subscribers and less initiated calls:

$$\begin{aligned}\frac{\partial \Pi_i}{\partial p_i} = & \underbrace{S_i D_i}_{\text{Call markup}} + \underbrace{\frac{\partial S_i}{\partial p_i}\frac{\Pi_i}{S_i}}_{\text{Loss of subscribers}} + \underbrace{S_i\left(p_i - S_i c - (1-S_i)(c_O + a)\right)\frac{\partial D_i}{\partial p_i}}_{\text{Fewer initiated calls}} \\ & + \underbrace{\frac{\partial S_i}{\partial p_i} S_i (a - c_T) D_i}_{\text{Composition effect}} - \underbrace{S_i \frac{\partial S_i}{\partial p_i}\left(a - c_T\right) D_j}_{\text{Marginal incoming calls}}.\end{aligned} \tag{3}$$

The first term on the last line constitutes a composition effect. As the number of subscribers goes down, more calls are terminated outside than inside the network. The composition effect is negative whenever it is more costly to terminate calls off-net than on-net ($a > c_T$). The final term is the marginal effect on termination profit resulting from a higher inflow of incoming off-net calls as the competitor's network becomes larger. Lost subscribers deliver termination profit (provided $a > c_T$) which serves to soften competition for subscribers. Increasing the subscription fee t_i has similar effects:

$$\begin{aligned}\frac{\partial \Pi_i}{\partial t_i} = & \underbrace{S_i}_{\text{Subsc. markup}} + \underbrace{\frac{\partial S_i}{\partial t_i}\frac{\Pi_i}{S_i}}_{\text{Loss of subscribers}} + \underbrace{S_i\left(p_i - S_i c - (1-S_i)(c_O + a)\right)\frac{\partial D_i}{\partial t_i}}_{\text{Fewer initiated calls (income effect)}} \\ & + \underbrace{\frac{\partial S_i}{\partial t_i} S_i (a - c_T) D_i}_{\text{Composition effect}} - \underbrace{S_i \frac{\partial S_i}{\partial t_i}\left(a - c_T\right) D_j}_{\text{Marginal incoming calls}}.\end{aligned} \tag{4}$$

A new term arises compared to the workhorse A-LRT model because the number of initiated calls falls as the subscription fee goes up ($\partial D_i/\partial t_i \leq 0$). This income effect depresses profit if the call price p_i exceeds the *perceived marginal call cost*, $S_i c + (1 - S_i)(c_O + a)$ - which is the average marginal on-net ($c = c_O + c_T$) and off-net ($c_O + a$) call cost weighted by relative call volumes inside and outside the network. By continuity, the existence and uniqueness results (Proposition 7) in Laffont, Rey and Tirole (1998a) generalize to the case of weak income effects:

Lemma 1 *Assume that each network charges a uniform price for calls. When the utility of subscribing to a network (v_0) is not too small, the degree of substitutability ($1/2\tau$) between the two networks is not too high, and the income effect (ε) is not too strong, there exists a unique and symmetric equilibrium. The equilibrium call price equals perceived marginal call cost, $P(a) = c + (a - c_T)/2$, and the equilibrium subscription fee $T(a)$ solves:*

$$\frac{T - f}{T} = \frac{1}{-\frac{\partial S_i}{\partial t_i} 2T} - \frac{1}{2}\frac{(a - c_T)}{T} D(c + (a - c_T)/2, T). \tag{5}$$

Proof: See the Appendix.

The network optimally sets the call price equal to perceived marginal cost so as to maximize the value of belonging to the network and uses the subscription fee to attract consumers. The subscription fee T satisfies a modified Lerner (inverse elasticity) rule. The equilibrium elasticity of subscription demand with respect to the subscription fee

$$-\frac{\partial S_i}{\partial t_i}2T = \frac{T}{\tau}\Lambda(c + (a - c_T)/2, T)$$

is a measure of the intensity of competition for subscribers. The lower is the elasticity of subscription demand, the higher is the equilibrium subscription fee, all else equal. Obviously, subscription elasticity is lower the stronger is the degree of network differentiation (the higher is τ), because then prices matter less for the choice of network. Second, the subscription elasticity is lower the lower is the marginal utility of income (Λ) because the subscription fee then is less important for consumer net surplus. The Lerner rule (5) is corrected by the composition effect. Setting a low subscription fee and gaining a high market share is extra profitable if off-net calls are more expensive than on-net calls ($a > c_T$) because the network then can save on call costs by having more calls terminated in the own network. The income effect enters into the equilibrium relation only indirectly via call demand. The direct income effect in (4) vanishes because calls are priced at marginal perceived call cost ($p_i = S_i c + (1 - S_i)(c_O + a)$).

The networks choose the reciprocal termination rate a to maximize industry profit, which under symmetry is equivalent to maximizing network profit

$$\pi(a) = \tfrac{1}{2}(T(a) - f) + \tfrac{1}{4}(a - c_T) D(c + (a - c_T)/2, T(a)), \tag{6}$$

which consists entirely of subscription profit and termination profit since initiated calls are priced at perceived marginal call cost. By agreeing on a higher termination rate, the two networks affect subscription profit and termination profit:

$$\pi'(a) = \tfrac{1}{2}T'(a) + \tfrac{1}{4}[D_j + (a - c_T)(\tfrac{1}{2}\tfrac{\partial D_j}{\partial p_j} + \tfrac{\partial D_j}{\partial t_j}T'(a))].$$

Each network runs a termination *deficit* whenever the termination rate lies below the marginal termination cost ($a \leq c_T$). If the subscription fee is increasing in the termination rate ($T'(a) \geq 0$), raising the termination rate unequivocally lowers the termination deficit ($\partial D_j/\partial p_j < 0$ and $\partial D_j/\partial t_j \leq 0$; see the Appendix) and simultaneously increases the subscription profit. Thus, setting a termination rate below marginal termination cost is profitable only if the subscription fee decreases sufficiently fast in the termination rate. Differentiation of the equilibrium subscription fee (5) yields

$$T'(a) = \frac{-\frac{1}{2}\frac{\partial}{\partial p_i}\left(-\frac{\partial S_i}{\partial t_i}2T\right) - \frac{1}{2}(D_j + \frac{1}{2}(a - c_T)\frac{\partial D_j}{\partial p_j})(\frac{\partial S_i}{\partial t_i}2T)^2}{(\frac{\partial S_i}{\partial t_i}2T)^2\left(1 + \frac{1}{2}(a - c_T)\frac{\partial D_j}{\partial t_j} + \frac{1}{4\tau}\frac{\partial \Lambda_i}{\partial t_i}(\frac{\partial S_i}{\partial t_i})^2\right)}.$$

Increasing the termination rate affects the subscription fee through two channels.[9] An anti-competitive effect pulls in the direction of a higher subscription fee. A higher termination rate means a higher call price and by implication lower call expenditures. Lower call expenditures imply a lower marginal utility of income and thereby a lower subscription elasticity:

$$\frac{\partial}{\partial p_i}\left(-\frac{\partial S_i}{\partial t_i}2T\right)\frac{\partial p_i}{\partial a}=\frac{1}{2}\frac{\partial \Lambda_i}{\partial p_i}\frac{T}{\tau}=\frac{1}{2}\frac{\varepsilon(\eta-1)U''\left(D_i\right)D_i}{\varepsilon p_i^2-U''\left(D_i\right)}\frac{T}{\tau}\leq 0.$$

Second, a higher termination rate reinforces the composition effect which tends to lower the equilibrium subscription fee. In the workhorse A-LRT model, subscription elasticity is independent of the termination rate (formally: $\Lambda = 1$ for $\varepsilon = 0$) and therefore the anti-competitive effect vanishes. Instead, the composition effect exactly offsets marginal termination profit, which renders profit independent of the termination rate. In the more general case of non-zero income effects, the anti-competitive effect is just big enough to pull in favour of high termination rates:

Proposition 1 *Assume that the conditions of Lemma 1 hold, so that there exists a unique and symmetric equilibrium. Then, network profit is independent of the termination rate if and only if the income effect is zero. In the presence of income effects, the networks can always raise profit by marginally increasing the termination rate (If $\varepsilon = (>)0$, then $\pi'(a) = (>)0$ for all a).*

Proof: See the Appendix.

Income effects have dramatic consequences for the choice of termination rate. Instead of being indifferent, the networks would always profit from increasing the termination rate. Of course, the termination rate cannot grow without bound. The upper bound to the termination rate arises at the point at which each network has an incentive to deviate and corner the market. Network competition with income effects presents a strong case for the regulation of termination rates and shows that the networks have a strong incentive for exaggerating production cost under cost-based regulation.

To gain additional insight into the mechanism driving profit neutrality, return to the A-LRT model, i.e. assume that there are no income effects. Let $v(a) = V(c + (a - c_T)/2, T(a))$ be consumer net surplus in symmetric equilibrium given the termination rate a. Define $\zeta(v(a)) = (\partial S_i/\partial v_i|_{v_1=v_2=v(a)})2v(a)$, the equilibrium subscription elasticity with respect to consumer net surplus. With quasi-linear preferences, the shadow price of the budget constraint equals unity ($\Lambda = 1$), and the equilibrium subscription fee solves:

$$T = f + \frac{v(a)}{\zeta(v(a))} - \frac{1}{2}\left(a - c_T\right)D(c + \left(a - c_T\right)/2).$$

[9] The denominator is strictly positive; see the Appendix.

Substituting the subscription fee above into consumer net surplus $v(a)$ and the profit function $\pi(a)$ yields:

$$v(a) = \frac{\zeta(v(a))}{1+\zeta(v(a))}W(a),\ 2\pi(a) = \frac{1}{1+\zeta(v(a))}W(a), \tag{7}$$

where

$$W(a) = U(D\left(c + (a - c_T)/2\right)) + I - cD\left(c + (a - c_T)/2\right) - f$$

is social surplus net of the utility of holding a subscription (v_0) and of the cost of horizontal differentiation ($\min\{k\tau; \tau(1-k)\}$). Social surplus is divided between the consumers and the industry in proportion to the subscription elasticity $\zeta(v(a))$. Most of the surplus goes to the consumers whenever subscription demand is elastic because of an intense competition for subscribers. Conversely, the networks extract most of the surplus under inelastic subscription demand because competition is weak in this case. The networks affect by their choice of termination rate both the size $W(a)$ of the social surplus to be divided and the share of that surplus the networks receive through the effect on competition $\zeta(v(a))$.[10] Under profit neutrality, the intensity of competition changes in exact proportion with social surplus. To see the fundamental property behind this result, divide $2\pi(a)$ by $v(a)$ in (7) and rewrite: $2\pi(a) = v(a)/\zeta(v(a))$. Obviously, profit neutrality holds if and only if equilibrium subscription elasticity is proportional to consumer net surplus, i.e. $\zeta(v(a)) = v(a)x$ for some $x > 0$ and for all a. The Hotelling model features proportional subscription demand at symmetric prices, $\zeta(v) = v/\tau$ for all v, and therefore profit neutrality follows.

More generally, all models in which market share is determined by the difference in consumer net surplus, $S_i = g(v_i - v)$, feature proportional subscription demand, $\zeta(v) = vg'(0)/g(0)$, and therefore profit neutrality: $2\pi(a) = g(0)/g'(0)$. The random utility model first used by Dessein (2003) for the duopoly case and later extended by Calzada and Valletti (2008) to the general $n \geq 2$ network case also belongs to the class of models with proportional subscription demand: $S_i = (1 + (n-1)e^{-\frac{1}{\gamma}\{v_i - v\}})^{-1}$ implies $\zeta(v) = v(n-1)/\gamma n$ and therefore $\pi(a) = \gamma/(n-1)$.[11]

Profit neutrality is a knife-edge result because it hinges on equilibrium subscription elasticity being exactly proportional to consumer net surplus. Introducing even a weak income effect breaks the proportionality and therefore profit neutrality. With income effects, social surplus grows faster than the intensity of competition for low termination rates, and so the profit maximizing termination rate is above the marginal cost of termination.

[10] The socially optimal choice of access charge is c_T when no income effects are present. Differentiate: $W'(a) = U'(D_i)\frac{1}{2}\frac{\partial D_i}{\partial p_i} - c\frac{1}{2}\frac{\partial D_i}{\partial p_i} = \frac{1}{4}(a - c_T)\frac{\partial D_i}{\partial p_i}$, where I have used $U'(D_i) = p_i = c + (a - c_T)/2$. Social surplus $W(a)$ is single-peaked in a and reaches its global maximum at c_T because $\partial D_i/\partial p_i < 0$ and $W'(c_T) = 0$.

[11] However, profit neutrality does not imply that subscription demand is a function of the differences $v_i - v$ in consumer net surplus. For example, $S_i = g((v_i/v)^v - 1)$ is proportional, $\zeta(v) = vg'(0)/g(0)$, but not a function of $v_i - v$.

4 Call Price Discrimination: The Off-Net Price Puzzle

The Model I now generalize the model in the previous section by allowing the networks to price discriminate between calls within the network (on-net) and calls outside the network (off-net). Price discrimination creates network externalities in the sense that the subscribers' choice of network now depends also on the size of the network and not only on prices. Every subscriber to network $i = 1, 2$ places q_i calls at the price p_i per call to every subscriber in the same network, and $\widehat{q}_i$ calls at the price $\widehat{p}_i$ per call to every subscriber in network $j \neq i$ to maximize utility $S_i U(q_i) + S_j U(\widehat{q}_i) + Z(y_i)$ and subject to the budget constraint $S_i p_i q_i + S_j \widehat{p}_i \widehat{q}_i + y_i + t_i \leq I$.

Utility maximization yields on-net demand $D_i = D(\mathbf{p}_i, t_i, S_i)$, off-net demand $\widehat{D}_i = \widehat{D}(\mathbf{p}_i, t_i, S_i)$, demand $Y_i = Y(\mathbf{p}_i, t_i, S_i)$ for the numeraire good and a shadow price of the subscription fee $\Lambda_i = \Lambda(\mathbf{p}_i, t_i, S_i)$ when all consumers have a subscription, $S_1 + S_2 = 1$, and $\mathbf{p}_i = (p_i, \widehat{p}_i)$ is the call-price profile of network i. Because of the income effect, on-net and off-net calls are substitutes, call demand decreases in the subscription fee and is ambiguous with respect to changes in the customer base; see the Appendix.[12] Define

$$u_i = u(\mathbf{p}_i, t_i, S_i) = U(D(\mathbf{p}_i, t_i, S_i)) - \Lambda(\mathbf{p}_i, t_i, S_i) p_i D(\mathbf{p}_i, t_i, S_i)$$

the indirect utility of reaching an on-net subscriber in network i, and let $\widehat{u}_i = \widehat{u}(\mathbf{p}_i, t_i, S_i)$ be the similarly defined indirect utility of reaching an off-net subscriber from network i. Consumer net surplus in network i is

$$\begin{aligned} v_i = V(\mathbf{p}_i, t_i, S_i) = \;& S_i u(\mathbf{p}_i, t_i, S_i) + (1 - S_i)\widehat{u}(\mathbf{p}_i, t_i, S_i) + Z(Y(\mathbf{p}_i, t_i, S_i)) \\ +\;& \Lambda(\mathbf{p}_i, t_i, S_i)(I - t_i - Y(\mathbf{p}_i, t_i, S_i)) \end{aligned} \tag{8}$$

when all consumers belong to one network or the other. Under the standard assumption of differentiated networks,

$$S_i = \alpha(V(\mathbf{p}_i, t_i, S_i), V(\mathbf{p}_j, t_j, 1 - S_i)) \tag{9}$$

uniquely defines subscription demand S_i in rational expectations equilibrium as a function of call prices $(\mathbf{p}_i, \mathbf{p}_j)$ and subscription fees (t_i, t_j). The profit of network i equals

$$\Pi_i = S_i[S_i (p_i - c) D_i + (1 - S_i)(\widehat{p}_i - a - c_O)\widehat{D}_i + t_i - f + (1 - S_i)(a - c_T)\widehat{D}_j].$$

The only difference between profit under price discrimination and uniform call prices is that off-net calls are priced at $\widehat{p}_i$ and $\widehat{p}_j$ instead of p_i and p_j.

Analysis By increasing the on-net price p_i, the network earns a higher markup per on-net call, but at the cost of a smaller number of subscribers and less initiated on-net calls per subscriber.

[12] In their analysis of residential call demand, Aldebert et al. (2004) report significant income elasticities as well as cross-price elasticities between local, national and international calls.

These effects constitute the three terms in the first line below:

$$
\begin{aligned}
\frac{\partial \Pi_i}{\partial p_i} = & \underbrace{S_i^2 D_i}_{\text{Call markup}} + \underbrace{\frac{\partial S_i}{\partial p_i}\frac{\Pi_i}{S_i}}_{\text{Loss of subscribers}} + \underbrace{S_i^2\left(p_i - c\right)\frac{\partial D_i}{\partial p_i}}_{\text{Fewer initiated on-net calls}} \\
& + \underbrace{\frac{\partial S_i}{\partial p_i}[(p_i - c)\, D_i - (\widehat{p}_i - a - c_O)\widehat{D}_i]}_{\text{Composition effect}} - \underbrace{S_i \frac{\partial S_i}{\partial p_i}\left(a - c_T\right)\widehat{D}_j}_{\text{Marginal incoming calls}} \\
& \underbrace{+ S_i \left[S_i\left(p_i - c\right)\frac{\partial D_i}{\partial S_i}\frac{\partial S_i}{\partial p_i} + S_j(\widehat{p}_i - a - c_O)\left(\frac{\partial \widehat{D}_i}{\partial p_i} + \frac{\partial \widehat{D}_i}{\partial S_i}\frac{\partial S_i}{\partial p_i}\right) - S_j\left(a - c_T\right)\frac{\partial \widehat{D}_j}{\partial S_j}\frac{\partial S_i}{\partial p_i}\right]}_{\text{Income effects}}.
\end{aligned} \tag{10}
$$

The first term on the second line is a composition effect, same as under uniform pricing: fewer subscribers means that relatively more calls are terminated off-net. The composition effect could be positive or negative depending on the profitability of on-net calls relative to off-net calls. The second term on the third line is the effect on termination profit of receiving more incoming calls. Increasing the on-net price generally affects demand for all types of calls through the budget constraint. The terms in the final lines characterize these income effects. Raising the off-net price $\widehat{p}_i$ and the subscription fee t_i have similar effects. For example:

$$
\begin{aligned}
\frac{\partial \Pi_i}{\partial t_i} = & \underbrace{S_i}_{\text{Subsc. markup}} + \underbrace{\frac{\partial S_i}{\partial t_i}\frac{\Pi_i}{S_i}}_{\text{Loss of subscribers}} + \underbrace{\frac{\partial S_i}{\partial t_i}[(p_i - c)\, D_i - (\widehat{p}_i - a - c_O)\widehat{D}_i]}_{\text{Composition effect}} - \underbrace{S_i \frac{\partial S_i}{\partial t_i}\left(a - c_T\right)\widehat{D}_j}_{\text{Marginal incoming calls}} \\
& \underbrace{+ S_i \left[S_i\left(p_i - c\right)\left(\frac{\partial D_i}{\partial t_i} + \frac{\partial D_i}{\partial S_i}\frac{\partial S_i}{\partial t_i}\right) + S_j(\widehat{p}_i - a - c_O)\left(\frac{\partial \widehat{D}_i}{\partial t_i} + \frac{\partial \widehat{D}_i}{\partial S_i}\frac{\partial S_i}{\partial t_i}\right) - S_j\left(a - c_T\right)\frac{\partial \widehat{D}_j}{\partial S_j}\frac{\partial S_i}{\partial t_i}\right]}_{\text{Income effects}}.
\end{aligned} \tag{11}
$$

By continuity, the existence and uniqueness results (Proposition 5) in Laffont, Rey and Tirole (1998b) generalize to the case of weak income effects:

Lemma 2 *Assume that both networks price discriminate between on-net and off-net calls. When the utility of subscribing to a network (v_0) is not too small, the degree of substitutability ($1/2\tau$) between the two networks is not too high, and the income effect (ε) is not too strong, there exists a unique and symmetric equilibrium. The equilibrium call prices equal marginal call cost: $P = c$ and $\widehat{P} = a + c_O$. The subscription fee satisfies:*

$$
\frac{T - f}{T} = \frac{1}{-\frac{\partial S_i}{\partial t_i} 2T} + \frac{1}{4}\frac{(a - c_T)}{T}\frac{\partial \widehat{D}_j(c, a + c_O, T, 1/2)}{\partial S_j}. \tag{12}
$$

Proof: See the Appendix.

The network optimally sets call prices at marginal call cost to maximize the social surplus inside the network and then uses the subscription fee to balance the loss of subscribers against surplus extraction. The optimal subscription fee satisfies a modified Lerner rule. The composition effect vanishes compared to the subscription fee (5) under uniform pricing: the network does not care about a larger fraction of initiated calls being terminated off-net when the markup on all initiated calls, on- and off-net, is zero. Instead, an expression related to termination profit shows up. In the presence of income effects, a higher market share of network j (S_j) affects demand for off-net calls in network j and therefore termination profit in network i. I show in the appendix that the second term is negative for all $a \neq c_T$, so reducing the subscription fee leads to a marginal increase in termination profit. In this sense, income effects intensify competition for subscribers under price discrimination, all else equal.

Just as was the case under uniform pricing, the subscription fee and termination profit are the sole sources of network profit

$$\pi(a) = \tfrac{1}{2}(T(a) - f) + \tfrac{1}{4}\,(a - c_T)\,\widehat{D}(c, a + c_O, T(a), 1/2)$$

because initiated calls are priced at marginal cost. The marginal effect on industry profit of increasing the reciprocal termination rate a thus equals:

$$\pi'(a) = \tfrac{1}{2}T'(a) + \tfrac{1}{4}[\widehat{D}_j + (a - c_T)\,(\tfrac{\partial \widehat{D}_j}{\partial \widehat{p}_j} + \tfrac{\partial \widehat{D}_j}{\partial t_j}T'(a))]. \tag{13}$$

Whether setting an termination rate below marginal termination cost is profitable depends on the sensitivity of the subscription fee to changes in the termination rate. If the subscription fee is non-decreasing in the termination rate ($T'(a) \geq 0$), it is profitable to increase the termination rate from any point below marginal termination cost ($a \leq c_T$) because then termination deficit falls and subscription profit increases. Only if the subscription fee falls sufficiently in the termination rate can it be profitable to set a termination rate below the marginal cost of termination. The key to understanding termination rates under call price discrimination therefore lies in exploring the sensitivity of the subscription fee to changes in the termination rate.

The equilibrium elasticity of subscription demand with respect to the subscription fee equals

$$-\frac{\partial S_i}{\partial t_i}2T = \frac{\Lambda_i T}{\tau - (u_i - \widehat{u}_i)} \tag{14}$$

under call price discrimination. As under uniform pricing, subscription elasticity is lower the stronger is the degree of network differentiation (the higher is τ) and the lower is marginal utility of income (Λ_i). Under call price discrimination, an additional *network multiplier* intensifies competition. A lower subscription fee means a higher market share, all else equal. A larger market share implies in turn that a larger fraction of every subscriber's calls are terminated on-net. If it is more valuable to connect with someone in the same network compared to someone in the other network ($u_i > \widehat{u}_i$) a higher market share further accentuates the benefit of belonging to that network. In the presence of network effects, there is a lot to gain in terms of extra subscribers by lowering the subscription fee because the flow of consumers multiplies itself. This process is faster the larger is the net benefit of on-net calls compared to off-net calls (measured by $u_i - \widehat{u}_i$).

Importantly, the networks affect competition for subscribers through the choice of termination rate because a higher off-net price lowers the marginal utility of income ($\partial\Lambda_i/\partial\widehat{p}_i \leq 0$) and strengthens the network effect ($\partial(u_i - \widehat{u}_i)/\partial\widehat{p}_i > 0$). The net effect is ambiguous in general and depends on the magnitude of the income effect and the degree of network differentiation:

$$\frac{\partial}{\partial\widehat{p}_i}\left(-\frac{\partial S_i}{\partial t_i}2T\right) = \frac{T\Lambda_i}{\widehat{p}_i(\tau - (u_i - \widehat{u}_i))}\left(\underbrace{\frac{\partial(u_i - \widehat{u}_i)}{\partial\widehat{p}_i}\frac{\widehat{p}_i}{\tau - (u_i - \widehat{u}_i)}}_{+} + \underbrace{\frac{\partial\Lambda_i}{\partial\widehat{p}_i}\frac{\widehat{p}_i}{\Lambda_i}}_{-/0}\right).$$

The elasticity of the network multiplier is small in the standard case of differentiated networks (when τ is high). Nonetheless, it dominates in the workhorse A-LRT model because the income effect there is zero ($\partial\Lambda_i/\partial\widehat{p}_i = 0$ for $\varepsilon = 0$). The networks then soften competition by choosing a low termination rate. The optimal termination rate lies below the marginal cost of termination (Gans and King, 2001). Even small income effects are enough to overturn this result, and render it profitable for the networks to agree on a termination rate above the marginal cost of termination:

Proposition 2 *Assume that the conditions of Lemma 2 hold, so that there exists a unique and symmetric equilibrium under call price discrimination. The profit maximizing termination rate lies below the marginal cost of termination if the income effect is zero. Then, the off-net price is lower than the on-net price. In the presence of income effects and if the networks are differentiated, there exists a termination rate above cost which generates higher network profit than any termination rate below cost. The off-net price then is higher than the on-net price (If* $\varepsilon = 0$*, then* $\pi(a^*) > \pi(a)$ *for some* $a^* < c_T$ *and for all* $a \geq c_T$*. Also* $P - \widehat{P} = c_T - a^* > 0$*. If* $\varepsilon > 0$*, but small, and* $\tau\varepsilon > 2/(\eta - 1)$*, then* $\pi(a^*) > \pi(a)$ *for some* $a^* > c_T$ *and for all* $a \leq c_T$*. Now,* $\widehat{P} - P = a^* - c_T > 0$*).*

Proof: See the Appendix.

The above results on optimal termination rates under uniform prices (Proposition 1) and under call price differentiation (Proposition 2) are derived under standard assumptions. The underlying assumption of differentiated networks is quite common in the literature because network differentiation allows a high degree of freedom in the choice of termination rates, while preserving concavity of the profit function and uniqueness of subscription demand. As Propositions 1 and 2 demonstrate, it then only takes a minor departure from the workhorse A-LRT model in terms of income effects to reverse the puzzling results and instead deliver results consistent with regulatory concern and the pricing policies the networks actually use.

5 Conclusion

Authorities remain sceptical to network competition despite extensive entry of competing networks over the last decades, recent years' market growth and the significant benefits telecommunications have brought to consumers and producers.[13] A main concern are the termination rates the operators charge for connecting calls from other networks. Excessive termination rates inflate the networks' perceived call costs, thereby distorting competition and prices. Owing to their adverse effect on consumer surplus, termination rates typically are regulated. A common requirement, at least in Europe, is that termination rates should not exceed estimated long run incremental cost.

Based upon the seminal contributions on network competition one would conclude that the prospects for efficient cost-based regulation are bright. The termination rate does not affect profit if the networks are also required to charge uniform call prices (Laffont, Rey and Tirole, 1998a). When the networks do not care about the termination rate, they also have no incentive to lie about cost under cost-based regulation. Cost-based regulation should thus implement the first-best welfare optimum.

In this paper I show that regulated networks under plausible assumptions may instead have a strong incentive to exaggerate their costs. The workhorse model of network competition (Armstrong, 1998; Laffont, Rey and Tirole, 1998a,b) rests on the assumption that changes in disposable income have no effect on call demand. In reality, call demand seems to display significant income effects; see e.g. Acton and Vogelsang (1992), Munoz and Amaral (1996), Aldebert et al. (2004). Income effects open up a channel through which excessive termination rates soften competition for subscribers. Higher termination rates spill over to higher call prices and lower call expenditures. Consumers who spend less on calls have a higher residual income and thereby a lower marginal utility of income. The lower is the marginal utility of income, the less important is the subscription fee for the consumers' choice of network and the softer is competition. Thus, higher termination rates imply softer competition in the presence of income effects. As profit neutrality is a knife-edge result, even an infinitesimal income effect softens competition enough to tilt networks to favour very high termination rates. Weak income effects are enough to render excessive termination rates profitable also under call price discrimination, provided the networks are sufficiently differentiated. Thus, networks have an incentive to overstate costs under cost-based termination regulation. A well designed regulatory policy should take this incentive to exaggerate costs into account.

Appendix

[13] In a sample of 21 OECD countries, Röller and Waverman (2001) attribute one third of economic growth over the period 1970-90 to telecommunications infrastructure investments.

Call demand

Uniform call prices Construct the Lagrangian $\mathcal{L}_i = U(q_i) + Z(y_i) + \lambda_i(I - t_i - p_i q_i - y_i)$, where λ_i is the Lagrangian multiplier associated with the budget constraint. Total differentiation of the first-order conditions $U'(D_i) - \Lambda_i p_i = 0$, $Z'(Y_i) - \Lambda_i = 0$ and the budget constraint $I - t_i - p_i D_i - Y_i = 0$ yield

$$\begin{bmatrix} U''(D_i) & 0 & -p_i \\ 0 & Z''(Y_i) & -1 \\ -p_i & -1 & 0 \end{bmatrix} \begin{bmatrix} dD_i \\ dY_i \\ d\Lambda_i \end{bmatrix} = \begin{bmatrix} \Lambda_i dp_i \\ 0 \\ D_i dp_i + dt_i \end{bmatrix}$$

under the assumption of a fully covered market, $S_1 + S_2 = 1$. Define the total call elasticity

$$\eta_i = -\frac{dD_i}{d(\Lambda_i p_i)} \frac{\Lambda_i p_i}{D_i} = -\frac{U'(D_i)}{U''(D_i) D_i}.$$

In Laffont, Rey and Tirole (1998a), $U(q) = (1-\eta^{-1})^{-1} q^{1-\eta^{-1}}$, which implies a constant elasticity $\eta_i = \eta > 1$. Apply Cramer's rule to the optimality conditions:

$$\begin{aligned} \frac{\partial D_i}{\partial p_i} &= \frac{\Lambda_i - Z''(Y_i) p_i D_i}{U''(D_i) + Z''(Y_i) p_i^2} < 0, & \frac{\partial D_i}{\partial t_i} &= \frac{-Z''(Y_i) p_i}{U''(D_i) + Z''(Y_i) p_i^2} \le 0, \\ \frac{\partial \Lambda_i}{\partial p_i} &= \frac{Z''(Y_i) U''(D_i)(\eta_i - 1) D_i}{U''(D_i) + Z''(Y_i) p_i^2}, & \frac{\partial \Lambda_i}{\partial t_i} &= \frac{-Z''(Y_i) U''(D_i)}{U''(D_i) + Z''(Y_i) p_i^2} \ge 0. \end{aligned} \qquad (15)$$

Call Price Discrimination Construct the Lagrangian

$$\mathcal{L}_i = S_i U(q_i) + S_j U(\widehat{q}_i) + Z(y_i) + \lambda_i(I - t_i - S_i p_i q_i - S_j \widehat{p}_i \widehat{q}_i - y_i),$$

where $\lambda_i \geq 0$ is the Lagrangian multiplier associated with the budget constraint.

The three first-order conditions $U'(D_i) - \Lambda_i p_i = 0$, $U'(\widehat{D}_i) - \Lambda_i \widehat{p}_i = 0$, $Z'(Y_i) - \Lambda_i = 0$, along with the budget constraint $I - t_i - S_i p_i D_i - S_j \widehat{p}_i \widehat{D}_i - Y_i = 0$ define four non-linear equations in the four unknowns $(D_i, \widehat{D}_i, Y_i, \Lambda_i)$. By total differentiation of the optimality conditions:

$$\begin{bmatrix} U''(D_i) & 0 & 0 & -p_i \\ 0 & U''(\widehat{D}_i) & 0 & -\widehat{p}_i \\ 0 & 0 & Z''(Y_i) & -1 \\ -S_i p_i & -S_j \widehat{p}_i & -1 & 0 \end{bmatrix} \begin{bmatrix} dD_i \\ d\widehat{D}_i \\ dY_i \\ d\Lambda_i \end{bmatrix} = \begin{bmatrix} \Lambda_i dp_i \\ \Lambda_i d\widehat{p}_i \\ 0 \\ S_i D_i dp_i + S_j \widehat{D}_i d\widehat{p}_i \\ +dt_i + (p_i D_i - \widehat{p}_i \widehat{D}_i) dS_i \end{bmatrix},$$

under the assumption of a fully covered market, $S_1 + S_2 = 1$. The determinant of the bordered Hessian is $-U''(D_i)\,U''(\widehat{D}_i)H_i$, where

$$H_i = 1 + \frac{Z''(Y_i)S_i(p_i)^2}{U''(D_i)} + \frac{Z''(Y_i)(1-S_i)(\widehat{p}_i)^2}{U''(\widehat{D}_i)} \geq 1.$$

By repeated application of Cramer's rule, the following comparative statics results are straightforward:

$$\begin{aligned}
&\frac{\partial D_i}{\partial p_i}U''(D_i)\,H_i = \Lambda_i - Z''(Y_i)(S_i p_i D_i + \eta_i S_j \widehat{p}_i \widehat{D}_i),\\
&\frac{\partial \widehat{D}_i}{\partial p_i}U''(D_i)\,H_i = \Lambda_i - Z''(Y_i)(S_i p_i D_i + \widehat{\eta}_i S_j \widehat{p}_i \widehat{D}_i),\\
&\frac{\partial \widehat{D}_i}{\partial p_i}\frac{U''(\widehat{D}_i)}{\widehat{p}_i}H_i = \frac{\partial \Lambda_i}{\partial p_i}H_i = Z''(Y_i)\,(\eta_i - 1)\,S_i D_i,\\
&\frac{\partial D_i}{\partial \widehat{p}_i}\frac{U''(D_i)\,H_i}{p_i} = \frac{\partial \Lambda_i}{\partial \widehat{p}_i}H_i = Z''(Y_i)(\widehat{\eta}_i - 1)S_j \widehat{D}_i,\\
&\frac{\partial \widehat{D}_i}{\partial \widehat{p}_i}U''(\widehat{D}_i)H_i = \Lambda_i - Z''(Y_i)(\eta_i S_i p_i D_i + S_j \widehat{p}_i \widehat{D}_i),\\
&\frac{\partial D_i}{\partial t_i}\frac{U''(D_i)\,H_i}{p_i} = \frac{\partial \widehat{D}_i}{\partial t_i}\frac{U''(\widehat{D}_i)H_i}{\widehat{p}_i} = \frac{\partial \Lambda_i}{\partial t_i}H_i = -Z''(Y_i),\\
&\frac{\partial D_i}{\partial S_i}\frac{U''(D_i)\,H_i}{p_i} = \frac{\partial \widehat{D}_i}{\partial S_i}\frac{U''(\widehat{D}_i)H_i}{\widehat{p}_i} = \frac{\partial \Lambda_i}{\partial S_i}H_i = Z''(Y_i)(\widehat{p}_i\widehat{D}_i - p_i D_i),
\end{aligned} \tag{16}$$

where $\eta_i = -U'(D_i)/U''(D_i)\,D_i$, and $\widehat{\eta}_i$ is similarly defined. In Laffont, Rey and Tirole (1998b), $\eta_i = \widehat{\eta}_i = \eta > 1$.

Proof of Lemma 1

Differentiate $S_i = \alpha(v_i, v_j)$ and apply the envelope theorem to v_i defined in (1):

$$\frac{\partial S_i}{\partial p_i} = \frac{\partial \alpha}{\partial v_i}\frac{\partial v_i}{\partial p_i} = -\frac{\partial \alpha}{\partial v_i}\Lambda_i D_i = \frac{\partial \alpha}{\partial v_i}\frac{\partial v_i}{\partial t_i}D_i = \frac{\partial S_i}{\partial t_i}D_i. \tag{17}$$

Subtract $(\partial \Pi_i/\partial t_i)D_i$ derived in (4) from $\partial \Pi_i/\partial p_i$ derived in (3) and use $(\partial S_i/\partial t_i)D_i = \partial S_i/\partial p_i$ derived above to get

$$\frac{\partial \Pi_i}{\partial p_i} - \frac{\partial \Pi_i}{\partial t_i}D_i = S_i\,(p_i - S_i c - (1-S_i)(c_O + a))\left(\frac{\partial D_i}{\partial p_i} - \frac{\partial D_i}{\partial t_i}D_i\right).$$

Assuming that $S_i > 0$, the right-hand side of this expression is positive for all $p_i < S_i c + (1 - S_i)(c_O + a)$ and negative for all $p_i > S_i c + (1 - S_i)(c_O + a)$ because

$$\frac{\partial D_i}{\partial p_i} - \frac{\partial D_i}{\partial t_i}D_i = \frac{\Lambda_i}{U''(D_i) + Z_i'' p_i^2} < 0,$$

where I have substituted in the comparative statics from (15). Therefore, the first-order conditions $\partial \Pi_i / \partial p_i = 0$ and $\partial \Pi_i / \partial t_i = 0$ are satisfied at $S_i > 0$ only if $P_i = S_i c + (1 - S_i)(c_O + a)$. At an interior optimum, therefore, initiated calls are priced at weighted marginal call cost. In symmetric equilibrium, $S_i = 1/2$, so $P_1 = P_2 = P(a) = c + (a - c_T)/2$.

Existence of a unique and symmetric equilibrium At $P_i = c + (1 - S_i)(a - c_T)$ and $\varepsilon = 0$, the profit function Π_i is strictly quasi-concave in t_i, the subscription fees are strategic complements and the reaction functions have a slope which is positive, but below unity; see Laffont, Rey and Tirole (1998a). By continuity, these properties extend also to the case with non-zero but weak income effects ($\varepsilon \gtrsim 0$). Hence, there exists a unique and symmetric equilibrium, provided v_0 is large, τ is large and ε is small. Given $P(a) = c + (a - c_T)/2$, the symmetric subscription fee solves the first-order condition

$$\frac{\partial \Pi_i}{\partial t_i} = 0 \Leftrightarrow T = f + \frac{1}{-2\frac{\partial S_i}{\partial t_i}} - (a - c_T)\, D(c + (a - c_T)/2, T)/2,$$

which can be rewritten on the form (5). ■

Proof of Proposition 1

If v_0 is large, τ is large and ε is small, but positive, the equilibrium subscription fee is given by (5). In the Hotelling model, $\partial S_i / \partial v_i = 1/2\tau$, see (2), so $-2\partial S_i / \partial t_i = \Lambda(p_i, t_i)/\tau$, see (17). Hence, the symmetric subscription fee in this case is the implicit solution to:

$$T = f + \tau / \Lambda(c + (a - c_T)/2, T) - (a - c_T)\, D(c + (a - c_T)/2, T)/2.$$

By implicit differentiation:

$$T'(a) = -\frac{1}{2} \frac{\left(\frac{\tau}{\Lambda_i^2} \frac{\partial \Lambda_i}{\partial p_i} + D_i + \frac{1}{2}(a - c_T) \frac{\partial D_i}{\partial p_i} \right)}{1 + \frac{1}{2}(a - c_T) \frac{\partial D_i}{\partial t_i} + \frac{\tau}{\Lambda_i^2} \frac{\partial \Lambda_i}{\partial t_i}},$$

which is of ambiguous sign. Plugging the expression for $T(a)$ into the equilibrium profit function (6), industry profit simplifies to $2\pi(a) = \tau / \Lambda(c + (a - c_T)/2, T(a))$. By substituting in the above expression for $T'(a)$:

$$\pi'(a) = -\frac{\tau}{2\Lambda_i^2} \left(\frac{1}{2} \frac{\partial \Lambda_i}{\partial p_i} + \frac{\partial \Lambda_i}{\partial t_i} T'(a) \right) = \frac{\tau}{4\Lambda_i^2} \frac{(D_i + \frac{1}{2}(a - c_T) \frac{\partial D_i}{\partial p_i}) \frac{\partial \Lambda_i}{\partial t_i} - (1 + \frac{1}{2}(a - c_T) \frac{\partial D_i}{\partial t_i}) \frac{\partial \Lambda_i}{\partial p_i}}{1 + \frac{1}{2}(a - c_T) \frac{\partial D_i}{\partial t_i} + \frac{\tau}{\Lambda_i^2} \frac{\partial \Lambda_i}{\partial t_i}}.$$

Note that

$$D_i + \frac{1}{2}(a - c_T)\frac{\partial D_i}{\partial p_i} = \frac{\Lambda_i (a - c_T) + 2U''(D_i) D_i + 2Z''(Y_i)cp_iD_i}{2\left(U''(D_i) + Z''(Y_i)p_i^2\right)}, \tag{18}$$

and

$$1 + \frac{1}{2}(a - c_T)\frac{\partial D_i}{\partial t_i} = \frac{U''(D_i) + Z''(Y_i)cp_i}{U''(D_i) + Z''(Y_i)p_i^2} > 0 \tag{19}$$

after substituting in the relevant expressions from (15), using $p_i = c+(a-c_T)/2$ and simplifying. The denominator of $\pi'(a)$ is strictly positive for all $\varepsilon \geq 0$ and all a by (19) and $\partial\Lambda_i/\partial t_i \geq 0$; see (15). Using (18) and (19) and substituting in the expressions for $\partial\Lambda_i/\partial t_i$ and $\partial\Lambda_i/\partial p_i$ from (15), the numerator of $\pi'(a)$ simplifies to (after utilizing also $p_i = c + (a - c_T)/2$):

$$\frac{\tau Z''(Y_i)U''(D_i)}{2(U''(D_i) + Z''(Y_i)p_i^2)^2}[-2\eta_i U''(D_i) D_i - \Lambda_i (a - c_T) - 2Z''(Y_i)\eta_i cp_iD_i].$$

The term in square brackets is strictly positive for all $\varepsilon \geq 0$ and all a. To see this, substitute in $\eta_i = -U'(D_i)/U''(D_i) D_i$, use the first-order condition $U'(D_i) = \Lambda_i p_i$ and $p_i = c + (a - c_T)/2$:

$$\begin{aligned} -2\eta_i U''(D_i) D_i - \Lambda_i (a - c_T) &= 2U'(D_i) - \Lambda_i (a - c_T) \\ &= 2\Lambda_i p_i - \Lambda_i (a - c_T) = 2\Lambda_i c > 0, \end{aligned}$$

and since $Z''(Y_i) \leq 0$, the result follows. For $\varepsilon = 0$, $Z''(Y_i) = 0$ and the numerator of $\pi'(a)$ equals zero for all a. This is profit neutrality. For $\varepsilon > 0$, $Z''(Y_i) < 0$, and the numerator of $\pi'(a)$ is strictly positive for all a. In this case, the networks can always profit from a marginally higher termination rate. ■

Proof of Lemma 2

Marginal cost pricing of initiated calls By total differentiation of (9) and an application of the envelope theorem to (8):

$$dS_i = -\frac{\frac{\partial\alpha}{\partial v_i}\Lambda_i(S_iD_idp_i + S_j\widehat{D}_id\widehat{p}_i + dt_i)}{1 - \frac{\partial\alpha}{\partial v_i}(u_i - \widehat{u}_i) + \frac{\partial\alpha}{\partial v_j}(u_j - \widehat{u}_j)}. \tag{20}$$

Take advantage of the fact that $\partial S_i/\partial p_i = (\partial S_i/\partial t_i)S_iD_i$ and $\partial S_i/\partial\widehat{p}_i = (\partial S_i/\partial\widehat{p}_i)S_j\widehat{D}_i$, subtract (11) from (10) and $\partial\Pi_i/\partial t_i$ from $\partial\Pi_i/\partial\widehat{p}_i$ to get:

$$\begin{aligned} \frac{\partial\Pi_i}{\partial p_i} - S_iD_i\frac{\partial\Pi_i}{\partial t_i} &= S_i^2 (p_i - c)\left(\frac{\partial D_i}{\partial p_i} - S_iD_i\frac{\partial D_i}{\partial t_i}\right) \\ &+ S_iS_j(\widehat{p}_i - a - c_O)\left(\frac{\partial\widehat{D}_i}{\partial p_i} - S_iD_i\frac{\partial\widehat{D}_i}{\partial t_i}\right) \end{aligned} \tag{21}$$

$$\begin{aligned}\frac{\partial \Pi_i}{\partial \widehat{p}_i} - S_j \widehat{D}_i \frac{\partial \Pi_i}{\partial t_i} &= S_i^2 (p_i - c) \left(\tfrac{\partial D_i}{\partial \widehat{p}_i} - S_j \widehat{D}_i \tfrac{\partial D_i}{\partial t_i} \right) \\ &+ \; S_i S_j (\widehat{p}_i - a - c_O) \left(\tfrac{\partial \widehat{D}_i}{\partial \widehat{p}_i} - S_j \widehat{D}_i \tfrac{\partial \widehat{D}_i}{\partial t_i} \right). \end{aligned} \tag{22}$$

Under the assumption of $S_i > 0$, the right-hand side of (21) is strictly negative if $p_i > c$ and $\widehat{p}_i \leq a + c_O$ and strictly positive if $p_i < c$ and $\widehat{p}_i \geq a + c_O$ because

$$\begin{aligned} \frac{\partial D_i}{\partial p_i} - S_i D_i \frac{\partial D_i}{\partial t_i} &= \frac{\Lambda_i - Z''(Y_i) \widehat{\eta}_i S_j \widehat{p}_i \widehat{D}_i}{U''(D_i) H_i} < 0 \\ \frac{\partial \widehat{D}_i}{\partial p_i} - S_i D_i \frac{\partial \widehat{D}_i}{\partial t_i} &= \frac{Z''(Y_i) \eta_i S_i p_i D_i}{U''(\widehat{D}_i) H_i} \geq 0, \end{aligned}$$

where I have used (16). At optimum $\partial \Pi_i / \partial p_i = \partial \Pi_i / \partial t_i = 0$, so $P_i \neq c$ is part of a profit maximizing two-part tariff only if $sgn\{P_i - c\} = sgn\{\widehat{P}_i - a - c_O\}$.

Add (21) and (22):

$$\begin{aligned} \frac{\partial \Pi_i}{\partial p_i} + \frac{\partial \Pi_i}{\partial \widehat{p}_i} - (S_i D_i + S_j \widehat{D}_i) \frac{\partial \Pi_i}{\partial t_i} &= S_i^2 (P_i - c) \left(\frac{\partial D_i}{\partial p_i} + \frac{\partial D_i}{\partial \widehat{p}_i} - (S_i D_i + S_j \widehat{D}_i) \frac{\partial D_i}{\partial t_i} \right) \\ &+ S_i S_j (\widehat{P}_i - a - c_O) \left(\frac{\partial \widehat{D}_i}{\partial p_i} + \frac{\partial \widehat{D}_i}{\partial \widehat{p}_i} - (S_i D_i + S_j \widehat{D}_i) \frac{\partial \widehat{D}_i}{\partial t_i} \right) = 0 \end{aligned}$$

at optimum. After some algebraic manipulations:

$$\frac{\partial D_i}{\partial p_i} + \frac{\partial D_i}{\partial \widehat{p}_i} - (S_i D_i + S_j \widehat{D}_i) \frac{\partial D_i}{\partial t_i} = \frac{\Lambda_i (1 + \varepsilon S_j \widehat{P}_i (P_i - \widehat{P}_i) / U''(\widehat{D}_i))}{U''(D_i) H_i}. \tag{23}$$

As shown by Laffont, Rey and Tirole (1998b), profit maximization implies $P_i = c$, $\widehat{P}_i = a + c_O$ for $\varepsilon = 0$. By continuity, $\lim_{\varepsilon \to 0} \varepsilon S_j \widehat{P}_i (P_i - \widehat{P}_i) / U''(\widehat{D}_i) = \lim_{\varepsilon \to 0} \varepsilon S_j (a + c_O)(c_T - a) / U'' \left(U'^{-1}(a + c_O) \right) = 0$. Thus, for ε small, but positive, profit maximization implies (23) negative. By an analogous argument, even $\partial \widehat{D}_i / \partial p_i + \partial \widehat{D}_i / \partial \widehat{p}_i - (S_i D_i + S_j \widehat{D}_i)(\partial \widehat{D}_i / \partial t_i)$ is negative at optimum for ε small. Thus, any equilibrium satisfying $P_i \neq c$ implies $sgn\{P_i - c\} = -sgn\{\widehat{P}_i - a - c_O\}$ for ε small but positive, which contradicts the necessary condition $sgn\{\widehat{P}_i - a - c_O\} = sgn\{P_i - c\}$, previously established. Thus, for ε small, but positive: $P_i = c$ and by implication also $\widehat{P}_i = a + c_O$.

Existence of a unique and symmetric equilibrium At $p_i = c$, $\widehat{p}_i = a + c_O$ and $\varepsilon = 0$, the profit function Π_i is strictly quasi-concave in t_i, the subscription fees are strategic complements and the reaction function has a slope which is positive, but below unity; see Laffont, Rey and Tirole (1998b). By continuity, these properties extend also to the case with non-zero, but weak income effects. Hence, there exists a unique and symmetric equilibrium, provided v_0 is large, τ is large and ε is small. Calls are priced at marginal cost, $P_i = c$ and $\widehat{P}_i = a + c_O$. The subscription fee solves the first-order condition

$$\frac{\partial \Pi_i}{\partial t_i} = 0 \Leftrightarrow T = f + \frac{1}{-2\frac{\partial S_i}{\partial t_i}} + \frac{(a - c_T)}{4} \frac{\partial \widehat{D}_j}{\partial S_j},$$

which can be rewritten on the form (12).

I now demonstrate that $(a - c_T)\left(\partial \widehat{D}_j / \partial S_j\right) < 0$ for all $a \neq c_T$ and $\varepsilon > 0$ at symmetric equilibrium. From the comparative statics results (16):

$$\frac{\partial \widehat{D}_i}{\partial S_i} = \frac{\widehat{p}_i Z''(Y_i)(\widehat{p}_i \widehat{D}_i - p_i D_i)}{U''(\widehat{D}_i) H_i}.$$

At symmetric prices, $\mathbf{p}_1 = \mathbf{p}_2 = \mathbf{p} = (p, \widehat{p})$ and $t_1 = t_2 = t$, $sgn\{\partial \widehat{D}_i / \partial S_i\} = sgn\{M(\mathbf{p}, t)\}$, where

$$M(\mathbf{p}, t) = \widehat{p}\widehat{D}(\mathbf{p}, t, 1/2) - pD(\mathbf{p}, t, 1/2)$$

equals the difference in call expenditures between off-net and on-net calls at symmetric prices. The marginal rate of substitution between on-net and off-net calls, $U'(D)/U'(\widehat{D}) = p/\widehat{p}$, and strict concavity of U imply $D = \widehat{D}$ whenever $p = \widehat{p}$, and therefore $M(p, p, t) = 0$. Moreover,

$$\begin{aligned} \frac{\partial M(\mathbf{p}, t)}{\partial \widehat{p}} &= \widehat{D} + \widehat{p}\frac{\partial \widehat{D}}{\partial \widehat{p}} - p\frac{\partial D}{\partial \widehat{p}} \\ &= -\frac{(\eta - 1)\widehat{D}}{\Lambda H}(\Lambda - Z''(Y)\eta p D) < 0 \end{aligned}$$

after simplifications. Therefore, $sgn\{\partial \widehat{D}_i / \partial S_i\} = sgn\{M(\mathbf{p}, t)\} = sgn\{p - \widehat{p}\}$ at symmetric prices. At symmetric equilibrium, $P - \widehat{P} = c_T - a$, so $sgn\{(a - c_T)\partial \widehat{D}_i / \partial S_i\} = sgn\{(a - c_T)(c_T - a)\} = -1$ for all $a \neq c_T$ and $\varepsilon > 0$ at symmetric equilibrium. ■

Proof of Proposition 2

First, some preliminaries. In the Hotelling model $\partial S_i / \partial t_i = -\Lambda_i / (2\tau + \widehat{u}_i - u_i + \widehat{u}_j - u_j)$; see (20). Hence, the symmetric subscription fee solves:

$$T = f + \frac{\tau + \widehat{u}(c, a + c_O, T, 1/2) - u(c, a + c_O, T, 1/2)}{\Lambda(c, a + c_O, T, 1/2)} + \frac{1}{4}(a - c_T)\frac{\partial \widehat{D}_j(c, a + c_O, T, 1/2)}{\partial S_j}.$$

By total differentiation of the subscription fee:

$$T'(a) = \frac{-\left(\tau + U(\widehat{D}_i) - U\left(D_i\right)\right)\frac{\partial \Lambda_i}{\partial \widehat{p}_i} - \Lambda_i^2 \widehat{D}_i + \frac{\Lambda_i^2}{4}\left(\frac{\partial \widehat{D}_j}{\partial S_j} + \left(a - c_T\right)\frac{\partial^2 \widehat{D}_j}{\partial S_j \partial \widehat{p}_j}\right)}{\left(\tau + U(\widehat{D}_i) - U\left(D_i\right)\right)\frac{\partial \Lambda_i}{\partial t_i} + \Lambda_i^2\left(1 - \frac{1}{4}\left(a - c_T\right)\frac{\partial^2 \widehat{D}_j}{\partial S_j \partial t_j}\right)}.$$

Zero income effects ($\varepsilon = 0$) If $\varepsilon = 0$, then $\Lambda_i = 1$, $\partial \Lambda_i / \partial \widehat{p}_i = \partial \Lambda_i / \partial t_i = 0$, $\partial \widehat{D}_j / \partial S_j = 0$ and $\partial^2 \widehat{D}_j / \partial S_j \partial \widehat{p}_j = \partial^2 \widehat{D}_j / \partial S_j \partial t_j = 0$, so $T'(a) = -\widehat{D}_i$, which implies

$$2\pi'(a) = \tfrac{1}{2}[\widehat{D}_j + (a - c_T)\tfrac{\partial \widehat{D}_j}{\partial \widehat{p}_j}] + T'(a) = \tfrac{1}{2}[(a - c_T)\tfrac{\partial \widehat{D}_j}{\partial \widehat{p}_j} - \widehat{D}_j] < 0 \text{ for all } a \geq c_T,$$

where I have used symmetry, $\widehat{D}_i = \widehat{D}_j$, and $\partial \widehat{D}_j / \partial \widehat{p}_j < 0$.

Non-zero income effects ($\varepsilon > 0$) Since $\partial \widehat{D}_j / \partial \widehat{p}_j < 0$ and $\partial \widehat{D}_j / \partial t_j \leq 0$, it is sufficient that $T'(a) \geq 0$ for all $a \leq c_T$ to render $2\pi'(a) > 0$ for all $a \leq c_T$, see (13). I need to show that $T'(a) > 0$ for ε sufficiently low and τ sufficiently large. Let $\tau = \overline{\tau}\varepsilon^{-1}$, where $\overline{\tau}(\eta - 1) > 2$. Recall, $\partial \Lambda_i / \partial \widehat{p}_i = -\varepsilon(\eta - 1)S_j \widehat{D}_i / H_i$ and $\partial \Lambda_i / \partial t_i = \varepsilon / H_i$. Plug into $T'(a)$ above to get

$$T'(a) = \frac{\left(\overline{\tau} + \varepsilon U(\widehat{D}_i) - \varepsilon U\left(D_i\right)\right)(\eta - 1)\widehat{D}_i - 2H_i \Lambda_i^2\left(\widehat{D}_i - \frac{1}{4}\frac{\partial \widehat{D}_j}{\partial S_j} - \frac{1}{4}\left(a - c_T\right)\frac{\partial^2 \widehat{D}_j}{\partial S_j \partial \widehat{p}_j}\right)}{2\left(\overline{\tau} + \varepsilon U(\widehat{D}_i) - \varepsilon U\left(D_i\right)\right) + 2H_i \Lambda_i^2\left(1 - \frac{1}{4}\left(a - c_T\right)\frac{\partial^2 \widehat{D}_j}{\partial S_j \partial t_j}\right)}.$$

By evaluation of the comparative statics (16), $\partial \widehat{D}_j / \partial S_j \to 0$, $\partial^2 \widehat{D}_j / \partial S_j \partial \widehat{p}_j \to 0$ and $\partial^2 \widehat{D}_j / \partial S_j \partial t_j \to 0$, as $\varepsilon \to 0$. Moreover, $H_i \to 1$, $\Lambda_i \to 1$ and $\widehat{D}_i \to U'^{-1}(a + c_O)$ as $\varepsilon \to 0$. Thus, $\lim_{\varepsilon \to 0} T'(a) = (\overline{\tau}(\eta - 1) - 2)U'^{-1}(a + c_O)/2(\overline{\tau} + 1) > 0$ in this case. ■

References

Acton, Jan P. and Ingo Vogelsang (1992): Telephone Demand over the Atlantic: Evidence from Country-Pair Data, *Journal of Industrial Economics* 40, 305-323.

Aldebert, Marc, Marc Ivaldi and Chantal Roucolle (2004): Telecommunications Demand and Pricing Structure: An Econometric Analysis, *Telecommunication Systems* 25, 89-115.

Armstrong, Mark (1998): Network Interconnection in Telecommunications, *Economic Journal* 108, 545-564.

———— (2002): The Theory of Access Pricing and Interconnection, in Martin E. Cave, Sumit K. Majumdar and Ingo Vogelsang (eds.) *Handbook of Telecommunications Economics Vol. 1*, Amsterdam: Elsevier, 295-384.

——— and Julian Wright (2009): Mobile Call Termination, *Economic Journal* 119, F270-F307.

Berger, Ulrich (2005): Bill-and-Keep vs. Cost-Based Access Pricing Revisited, *Economics Letters* 86, 107-112.

Calzada, Joan and Tommaso M. Valletti (2008): Network Competition and Entry Deterrence, *Economic Journal* 118, 1223-1244.

Carter, Michael and Julian Wright (2003): Asymmetric Network Interconnection, *Review of Industrial Organization* 22, 27-46.

Dessein, Wouter (2003): Network Competition in Nonlinear Pricing, *RAND Journal of Economics* 34, 593-611.

——— (2004): Network Competition with Heterogeneous Customers and Calling Patterns, *Information Economics and Policy* 16, 323–345.

Gabrielsen, Tommy S. and Steinar Vagstad (2008): Why is On-net Traffic Cheaper than Off-net Traffic? Access Markup as a Collusive Device, *European Economic Review* 52, 99-115.

Gans, Joshua S. and Stephen P. King (2001): Using 'Bill and Keep' Interconnect Arrangements to Soften Network Competition, *Economics Letters* 71, 413-420.

Genakos, Christos D. and Tommaso Valletti (2009): Testing the "Waterbed" Effect in Mobile Telephony, *Journal of the European Economic Association*, forthcoming.

Hahn, Jong-Hee (2004): Network Competition and Interconnection with Heterogenous Subscribers, *International Journal of Industrial Organization* 22, 611-631.

Hoernig, Steffen, Roman Inderst and Tommaso Valletti (2009): Calling Clubs: Network Competition with Non-Uniform Calling Patterns, manuscript.

Hurkens, Sjaak and Ángel L. López (2010a): Mobile Termination, Network Externalities, and Consumer Expectations, manuscript.

——— (2010b): Mobile Termination and Consumer Expectations under the Receiver-Pays Regime, NET Institute WP #10-12.

Jeon, Doh-Shin, Jean-Jacques Laffont and Jean Tirole (2004): On the "Receiver-Pays" Principle, *RAND Journal of Economics* 35, 85-110.

Jullien, Bruno, Patrick Rey and Wilfried Sand-Zantman (2010): Mobile Call Termination Revisited, IDEI WP 551.

Laffont, Jean-Jacques, Patrick Rey and Jean Tirole (1998a): Network Competition: Overview and Nondiscriminatory Pricing, *RAND Journal of Economics* 29, 1-37.

——— (1998b): Network Competition: Price Discrimination, *RAND Journal of Economics* 29, 38-56.

López, Ángel L. and Patrick Rey (2009): Foreclosing Competition through Access Charges and Price Discrimination, IDEI Working Paper 570.

Muñoz, Teresa G. and Teodosio P. Amaral (1996): Demand for International Telephone Traffic in Spain: An Econometric Study Using Provincial Panel Data, *Information Economics and Policy* 8, 289-315.

OECD (2009): OECD Communications Outlook 2009.

PTS (2010): Telecommunication Markets in the Nordic Countries, downloaded from www.pts.se/sv/Dokument/Rapporter/Telefoni/2010/Telecommunication-Markets-in-the-Nordic-Countries, November 16, 2010.

Röller, Lars-Hendrick and Leonard Waverman (2001): Telecommunications Infrastructure and Economic Development: A Simultaneous Approach, *American Economic Review* 91, 909-923.

Valletti, Tommaso M. and Carlo Cambini (2005): Investments and Network Competition, *RAND Journal of Economics* 36, 446-467.

Vogelsang, Ingo (2003): Price Regulation of Access to Telecommunications Networks, *Journal of Economic Literature* 41, 830-862.