The dynamics of protection and imitation of innovations

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Abstract

A large portion of innovators do not patent their inventions. This is a relative puzzle for those who view innovators as at the mercy of imitators in the absence of legal protection. In practice innovators however invest actively in making their products technologically hard to reverse engineer. We consider the dynamics of imitation and protection of innovations. We show that innovators can obtain high profits in the absence of legal protection. Surprisingly, in general, the protection technologies that yield the highest profits for the innovator are expensive and do not protect well. Our model also allows us to draw conclusions on the design of patent policy and on the dynamics of employment and mobility of researchers in innovative industries.

1 Introduction

It is now well established that a large portion of innovations are not protected by patents. Some subject matters are still not patentable, although those are now few, but more importantly a substantial number of innovators choose other means of protection, such as secrecy. Moser (2011) shows that most innovations presented at 19th Century World Fairs were not patented.[1] Several influential surveys of managers (Cohen, Nelson and Walsh (2000), Arundel (2001)) document the fact that patents are rarely the most popular mean of appropriating returns from $R\&D$ investments. Based on a survey run in 1994, Cohen, Nelson and Walsh (2000) establish that only 34.8 percent of managers judged patents as an effective way of appropriating returns from product innovations (compared to 51 percent for secrecy).

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[1]89 percent for the 1851 fair.

This remains somewhat of a puzzle for many since the sources of profits for innovators outside patents are still badly understood. Most view successful innovators as helplessly at the mercy of aggressive imitators in the absence of legal protection. Some do acknowledge that imitation might be a process that takes time, and thus make secrecy relatively more attractive, but this lag between innovation and imitation is most often considered as exogenously given, a characteristic of the product or of the sector for instance. We believe on the contrary that innovators are not as helpless as commonly argued and in particular can control the speed of imitation by making their technologies harder to reverse engineer. The dynamics of protection will be the focus of our work.

Examples abound of firms pursuing protective measures beyond patent law. Ichijo (2010) illustrates this for some consumer electronics products: "Sharp has put tremendous efforts into making imitation of its LCD TV sets time consuming and difficult. Various initiatives at Kameyama are aimed at increasing complexities (...) in order to make imitation difficult". A similar behavior can also be observed in some high-tech manufacturing industries. The software industry is full of obfuscation strategies and tools designed to interfere with reading of the machine code or its decompilation. Not only software, but also hardware can be actively protected. For example, it is quite typical in the semiconductor industry to encase some of the important circuitry in epoxy blocks so that electronics are destroyed if someone tries to open them.[2] It is not unusual either to design the integrated circuits to have pieces that are seemingly unused but are required for the operation.

In this paper, we show that accounting for the dynamics of investments in such protective measures may radically affect our views on sources of profits for innovators outside patents and can also inform the design of the patent system. Dynamics are essential to approach this question. The common view is that free riding by imitators will be extremely harmful. However, this ignores two important facts. First, free riders upon imitation find themselves in a similar situation as the innovator, thus endogenously keeping barriers to entry high. Second, free riders can free ride on each other: if it is anticipated that the next imitator to enter will do so without paying for protection, all imitators delay their entry in the hope of benefiting from the efforts of one of their competitors.

These ideas are formalized in an infinite horizon model in which the inventor faces a pool of ex ante identical imitators who are initially inactive. At every period, those who have not yet reversed engineered the invention decide whether to do so at some (possibly low) imitation cost c_i. If they do, they also decide whether or not to pay a one-time protection cost c_p. If all previous entrants have paid c_p, the cost of reverse engineering for the remaining imitators is c_i. If at least one of them has not invested in

[2] Another common way to reverse engineer electronics and circuits is to use x-ray images and work out what components have been used. For this reason, firms try to hinder these imitation efforts by positioning parts in such a way that the x-ray recognition is hampered.

protection by paying c_p, the innovation becomes freely available. Protection technologies are characterized by their cost, c_p, and the strength they confer, c_i.

We focus on the case that in principle is the most unfavorable to the inventor: imitation is always profitable and can take place almost instantaneously.[3] We find in this setting that inventors can earn very high rents even in the absence of legal protection. Such high rents may also be well above those attained by imitators. Surprisingly, the protection technologies that tend to yield a high payoff for the inventor are expensive and do not protect very well (high c_p and small c_i). The intuition of this result is the following: the fact that the protection technology is expensive means that, upon entry, imitators will not use it. Thus, when the first imitator enters, the knowledge necessary to reproduce the product enters the public domain and all remaining imitators enter for free. This creates a strong incentive for imitators to free ride on other imitators' efforts, initially delay costly entry, thus leaving potentially very high profits to the innovator.

We also find that, for other protection technologies, the payoff to the inventor can be quite low. Our theory can thus help us understand the relative importance of secrecy and patenting across sectors. Indeed if the type or protection technologies available to firms varies across sectors, our theory predicts that in sectors where c_p is high and c_i small, patents should be less popular. Empirical evidence establishes that propensity to patent varies widely across sectors. However we do not currently have data on protection technologies, be it their cost or the extent of protection they confer. Interestingly though, according to surveys of managers (Cohen et al. 2000), the sectors for which examples of protection technology come most readily to mind, electronic components and semiconductors, are amongst the sectors where patents are judged to be least effective.[4]

We focused in our previous discussion on the most profitable protection technology, but we characterize in the paper the symmetric mixed-strategy equilibria for arbitrary technologies. This leads us to characterize a theoretically interesting pattern where typically a series of preemption game is followed with some probability by a war of attrition. Imitators are involved in a series of preemption games taking place quasi-instantaneously at the outset of the game. All the imitators that happen to enter pay the protection cost, but fear mis-coordination and thus mix at each instant between waiting and imitating. They pursue protection in the hope of securing some rents, anticipating that the initial phase of massive entry will be followed with some probability by a waiting game played by the imitators left to enter. Such a game involving delayed imitation arises because once a sufficient number of imitators have entered, the protection cost is too large relative to the post-entry payoff and the next imitator to enter does so without paying for protection. These imitators thus engage in a war of attrition delaying entry in the hope

[3] Other cases are considered in Section 4 of the paper.

[4] 21.3 percent for electronic components and 26.7 for semiconductors. The software industry is not part of the survey.

that a competitor enters before them.

Not only does our model provide strong predictions on the sources of inventor's profits in the absence of legal protection, but it can also be very informative on the design of patent policy. Indeed, a patent is a specific example of a protection technology: c_p is the cost of applying for the patent, while c_i measures the strength of the protection it confers. Unlike previous literature that has mostly focused on c_i (a rough proxy for patent breadth), our paper underscores that the cost of applying for a patent becomes an important policy lever once dynamics are accounted for. The cost of the patent influences both the decision of the inventor to initially apply but also the decision of the imitators who invent around the patent to themselves in turn apply. The relationship between patent fees and the inventor's profit is highly nonmonotonic, but our model suggests, surprisingly, that an increase in patent fees may lead to a higher payoff for the inventor.

We also examine a different dimension of protection that appears essential in practice. Knowledge is often diffused through scientists'mobility.[5] Investing in protection can be seen in this light as paying researchers sufficiently high wages so as to prevent them from leaving the firm. We capture this idea in a variant of our model where we add competition for the researcher following imitation. We show that the results are very close to the results of our initial model but renders the cost of protection endogenous. Furthermore we describe the mobility of researchers and their associated salaries.

Our paper is related to the growing literature on the sources of profits for innovators in the absence of intellectual property rights (e.g. Boldrin and Levine (2007, 2008, 2010), Maurer and Scotchmer (2002), Henry and Ponce (2009)). Boldrin and Levine (2007, 2008) present theoretical justification and practical examples of innovation flourishing in the absence of legal protection. Henry and Ponce (2009) show that the classical justification for patent protection is challenged if the innovator can license the knowledge necessary to reproduce the innovation. In our paper we do not consider licensing, but we revisit the canonical model by introducing the possibility, common in many industries, to invest in protection technologies. We also find that this can also change radically the results.

Our model also allows us to draw conclusions on the choice between patents and secrecy. There is a growing literature on this question, building on the seminal paper by Horstmann et al. (1985), who consider the signalling value of patents when innovators have private information on the value of imitation for competitors. Kultti et al.(2007) consider the comparison in a setting with multiple independent discoveries and where, under secrecy, the idea becomes public with a certain exogenous probability. Anton and Yao (2004) is closer to our work in the sense that the innovator can take strategic actions to decrease competition even when she chooses secrecy. Indeed, the innovator can choose a level of disclosure: more disclosure signals a better innovation and thus makes the

[5] Almeida and Kogut (1999) show for instance that scientific references cited in patent applications reflect the employment histories of the researchers.

imitator less aggressive in the competition stage. In our paper the strategy available to the innovator to deter imitation is of a different kind and furthermore we insist on the importance of dynamics.

Our paper also contributes to the literature on entry games with an infinite horizon of play. Our game exhibits a theoretically interesting pattern of a series of preemption games followed by a waiting game. Our approach towards analyzing continuous-time preemption games builds upon Fudenberg and Tirole (1985), except that we have more than two players (possibly) mixing over more than two actions.[6] The existence of equilibrium coordination failures directly relates our work to that of Dixit and Shapiro (1986), Cabral (1993, 2004), Vettas (2000) and Bertomeu (2009). Vettas (2000) is of particular relevance because he finds the remarkable result that the (continuation) payoff expected by an incumbent conditional upon entry is nonmonotonic in the number of firms active in the market. A similar nonmonotonicity result is derived in our setting, even though we allow flow profits to strictly decrease in the number of firms active in the market, unlike Vettas (2000). Our focus on continuous time allows us to dispense with his assumption, showing that his insights carry over to settings in which decisions can be made very often.

We conclude the relationship of our paper with past literature by observing that it is not usual to find entry timing games that display both preemption and waiting motives as play unfolds. Important exceptions are Sahuguet (2006) and Park and Smith (2008). Our preemption-then-attrition result resembles that in Sahuguet's (2006) analysis of volunteering for heterogeneous tasks under incomplete information about others' preferences. Besides our focus on complete information, we differ from his analysis in several other dimensions, especially in the questions analyzed (i.e., public good provision vs. innovation protection and imitation).

The remainder of the paper is organized as follows. In Section 2 we introduce the model. In Section 3 we solve for the equilibrium entry and protection decisions. In Section 4 we draw conclusions on the level of profits of the innovator in the absence of patents and discuss specific examples. In Section 5 we discuss the implications for patent policy while section 6 examines protection of knowledge through restraining worker mobility. All proofs are presented in the appendix.

[6] In addition, players in our setting fail to coordinate their actions with positive probability along the equilibrium path, whereas coordination failures only occur in their setting out of the equilibrium path. In both settings, players randomize in continuous time in a non-independent manner for reasons that can be already found in Fudenberg and Tirole (1985). In their framework, though, there exists a point in time at which players have an incentive to perfectly correlate their entry actions when mixing. In ours, there is no such a point because players always face the temptation of possibly profitable preemption from the very beginning of the game.

2 Model

We analyze a discrete-time game that lasts infinitely many periods of length $\Delta > 0$. The time variable is denoted by $t = 0, \Delta, 2\Delta, ...$ All players have the same per-period discount factor δ^{Δ}. We will focus on the case in which Δ is positive but converges to zero, i.e., the continuous-time limit of the game.

The game involves one innovator and $n-1 \geq 2$ (ex ante identical) potential imitators. Prior to the start of the game, the innovator has discovered a new technology. The imitators can then decide in each period whether to imitate or stay out of the market an additional period. We consider the dynamics of imitation of this technology. The cost of imitation depends on the strategic choices made by the innovator and the imitators who previously entered. In any period t, we call the imitators who have already imitated and entered the market the insiders, whereas we call the imitators who have not entered yet the outsiders.

The innovator at time $t = 0$ and the imitators upon entry need to decide whether to invest in protection. Protection technologies are characterized by two parameters c_i and c_p (see Subsection 4.3 for examples). We denote $c_p > 0$ for the one-time cost that needs to be incurred to achieve protection. In any period, if the innovator and all insiders incurred the protection cost $c_p > 0$ upon entry, the outsiders who decide to enter need to incur imitation cost $c_i > 0$.[7] This one-time cost c_i gives instantaneous access to the same technology. However, if one of the insiders did not pay c_p upon entry, then imitation becomes costless for all outsiders. We assume that the costs c_p and c_i remain fixed throughout the game, in particular they are independent of the number of firms active in the market.

In each period, an outsider can therefore choose among three actions:

- to imitate and pay the protection cost, an action denoted p
- to imitate and not pay the protection cost, an action denoted u
- not to imitate and wait another period, an action denoted w

Per-period profits depend on the number of firms that have entered. We denote π_j for the per-period individual profit if $j \in \{1, ..., n\}$ firms (including the innovator) hold the technology.[8] Denoting the rate at which profits are discounted by firms by r, let $\Pi_j \equiv \pi_j / r$ represent the value of a perpetual stream of discounted profits collected by a firm when a total of $j \in \{1, ..., n\}$ hold the technology and no further entry takes place.

[7] The assumptions on the protection and imitation costs are somewhat reminiscent of Bernheim (1984).

[8] To avoid introducing several effects that would obscure the message of the paper, we assume that the flow profits earned do not depend on whether the protection cost was incurred or not. In other words, making the technology harder to reverse engineer does not directly affect the willingness to pay of consumers or production costs.

We mostly focus, in particular in Section 3, on the case where $\Pi_n > c_i$. This corresponds to a situation where all firms will eventually enter the market: even if $n-1$ firms are already on the market and all the insiders and the innovator paid the protection cost c_p, imitation is still profitable. Note that this is a priori the worst case scenario for innovation in the absence of legal protection since the protection technology does not offer much of a guarantee.

We allow for mixed strategies and focus on symmetric Markov Perfect Equilibria (MPE), where the state corresponds to the number of firms that hold the technology. Markov perfection hardly needs any discussion given its overwhelming use in dynamic games in which collusion is not the aspect to analyze. The focus on symmetric (mixed-strategy) equilibria probably seems more restrictive. As Farrell and Saloner (1988) and Bolton and Farrell (1990) convincingly argue, though, decentralized coordination mechanisms involving anonymous players cannot be properly captured by asymmetric equilibria in which (asymmetric) roles are very well defined among players. In addition, play based on mixed strategies can be interpreted as play arising in a game in which each player has private information about some disturbance affecting her final payoff.[9] As pointed out by Cabral (1993), coordination failures occur under this interpretation not because of randomization but because players have incomplete information about others' payoffs.

Given our restriction on Markovian play, we use the following notation throughout: at the start of a period with k outsiders left to enter, we denote

- the expected discounted profits of an insider by I_k
- the expected discounted profits of an outsider if she decides to enter by O_k

3 The dynamics of protection and imitation

In this section we solve for the equilibria in the most interesting case where $\Pi_n > c_i$ (we consider the other case in Section 4). To help the reader through the arguments, we first sketch the shape of the equilibrium. The finiteness of the pool of potential imitators allows us to use backward induction when solving the infinite-horizon game, so we explain the reasoning by working backwards as well.

In the final subgames, when many firms are already active, the protection cost c_p appears large compared to the expected profits that can be reaped. The next entrant will thus enter without any protective measure, thereby creating an incentive for the remaining imitators to delay imitation in the hope of free riding on the next entrant. We actually find a critical number of outsiders J such that if the number of outsiders is strictly less than J, they play a war of attrition to determine who is free ridden by others.

[9] This is the well-known purification argument in Harsanyi (1973).

In earlier subgames with at least J outsiders, there is an incentive for imitators to enter quickly, preempt the others by protecting their technologies and benefit from the subsequent imitation delay. However, there is a risk of miscoordination where all imitators would enter simultaneously. This creates the conditions for a preemption game. In such a game at least one outsider will enter right away (and several could in fact enter simultaneously). If after entry the number of outsiders is still less than J, another preemption game is played, and so on and so forth until the number of active firms exceeds J. Overall, we see that the pattern is a series of preemption games followed by a war of attrition. Below, we make these arguments formal.

3.1 Solving the subgames with few outsiders

We note that in any subgame in which at least one of the insiders did not pay the protection cost upon entry, all outsiders immediately imitate the technology at no cost. Thus in the following discussion, we exclusively focus on subgames in which all firms who hold the technology paid c_p upon entry.

The last entrant

We begin our analysis by considering those subgames in which just one imitator is left to enter the market. Since $\Pi_n > c_i$, the last outsider enters immediately. The expected profit of an insider in such a subgame is $I_1 = \Pi_n$. The expected profit of the outsider is $O_1 = \Pi_n - c_i$ if all insiders and the imitator paid the protection cost, and $O_1 = \Pi_n$ otherwise.

Two imitators left to enter

We now consider the subgames with only two outsiders. The first outsider needs to incur cost c_i in order to enter. He knows that, regardless of whether or not he pays the additional protection cost, the remaining outsider will enter immediately. It is then clear that action p (entering and paying the protection cost) is strictly dominated.

Therefore, the first entrant does not choose protection, and the second entrant will incur a zero cost to imitate. This creates the conditions for a war of attrition where both players mix between entering without paying the protection cost and waiting.[10] Both players prefer to be the second entrant, but also do not want to wait excessively as they lose profits every period. As is standard in such games, in the limit when Δ converges to zero, the entry time of each imitator converges to an exponential distribution.

[10] As usual in dynamic games involving mixed strategies, it is more convenient to deal with behavioral strategies.

Lemma 1 *In subgames with two outsiders, the only symmetric MPE is such that both imitators mix between actions u (entering without paying the protection cost) and w (waiting another period). As* Δ *converges to zero, the entry time of each imitator converges to an exponential distribution with hazard rate* $\lambda_2 = r(\Pi_n - c_i)/c_i$*. The expected profit of the outsiders is* $O_2 = \Pi_n - c_i$*, whereas each of the insiders expects to gain* $I_2 = \mu_2 \Pi_{n-2} + (1 - \mu_2)\Pi_n$*, where* $\mu_2 \equiv r/(r + 2\lambda_2)$*.*

The expected payoff of an outsider is $O_2 = \Pi_n - c_i$ since he is indifferent between all entry times, including entering immediately. On the contrary, the insiders expect significant profits since they will earn per-period profit π_{n-2} until the time of first entry, which is exponentially distributed (with hazard rate $2\lambda_2$).

Three imitators left to enter

Before studying the complete dynamics, it is useful to understand in detail the resolution of subgames with three outsiders left. All players know that in any period, if a single outsider enters and pays the protection cost, then the remaining two imitators will play a war of attrition with speed of entry λ_2. In such a game, we established in Lemma 1 that insiders earn expected profits of $I_2 = \mu_2 \Pi_{n-2} + (1 - \mu_2)\Pi_n$.

Thus, we first note that, if $I_2 - c_p \leq \Pi_n$, playing action p is (weakly) dominated by u, that is, outsiders will never pay the protection cost. The conditions can be equivalently expressed as $c_p \geq c_2^* \equiv \mu_2(\Pi_{n-2} - \Pi_n)$. According to the same logic as in the previous section, the three imitators will then play a war of attrition. We show in Lemma 2 that the individual entry time then follows an exponential distribution of parameter $\lambda_3 \equiv r(\Pi_n - c_i)/(2c_i)$.

On the contrary, if $c_p < c_2^*$, preemptively entering and paying the protection cost becomes very attractive if the two other outsiders do not enter. There is however a risk of coordination failure where all outsiders simultaneously enter and pay c_p. This creates the conditions for a preemption game described in Lemma 2. Outsiders mix between p and w, that is, between entering and paying the protection cost and waiting. Entry occurs almost instantaneously with probability one, and simultaneous entry of several outsiders occurs with positive probability.

Lemma 2 *In subgames with three outsiders, as* Δ *converges to zero:*

(i) If $c_p \geq c_2^*$*, the three outsiders mix between actions u and p. Counting from the date at which the subgame is first reached, the individual distribution of entry times is exponentially distributed with parameter* λ_3*, where* $\lambda_3 \equiv r(\Pi_n - c_i)/(2c_i)$*. Furthermore,* $O_3 = \Pi_n - c_i$ *and* $I_3 = \mu_3 \Pi_{n-3} + (1 - \mu_3)\Pi_n$*, where* $\mu_3 \equiv r/(r + 3\lambda_3)$*.*

(ii) If instead $c_p < c_2^*$*, the three outsiders start playing a preemption game as soon as this subgame is reached. The limiting distribution is such that imitators play w and p*

with strictly positive probability, and the payoff of the outsiders converges to $O_3 = \Pi_n - c_i$*, whereas the payoff of the insiders converges to* $I_3 = \phi_3(1)I_2 + (1-\phi_3(1))\Pi_n$*, where* $\phi_3(1)$ *is the probability of a single outsider entering.*

Lemma 2 has a very natural interpretation. If the protection cost is relatively high, it will not be paid upon entry, and therefore all outsiders wait in the hope that one of them will move first. On the contrary, if the protection cost is low, it will be incurred upon entry. The problem is then one of coordination. All outsiders would like to be the only firm to enter and then enjoy payoff I_2 while the others play a war of attrition later on, but no one has an interest in paying the protection cost if other outsiders choose to enter immediately.

3.2 Subgames with more than three imitators left to enter

The ideas uncovered in the subgames with three outsiders partially extend to the subgames with a larger number of outsiders. In particular, if c_p is relatively large, the players will end up playing a war of attrition.

For $2 \leq k \leq n-1$, let $c_k^* \equiv \mu_k(\Pi_{n-k} - \Pi_n)$ from now on. We show in the following lemma that in the subgame with $k \geq 3$ outsiders, if $c_p \geq c_{k-1}^*$, players mix between waiting and entering without protection and the entry time is exponentially distributed. A key part of the induction argument is that $\{c_k^*\}_{k=2}^{n-1}$ is a monotonically increasing sequence.[11] This implies that, when $c_p \geq c_{k-1}^*$, if one outsider chooses to enter by paying the protection cost, the $k-1$ remaining outsiders would then play a war of attrition since $c_p > c_{k-2}^*$. Intuitively, the incentive to avoid paying the protection cost becomes more intense as fewer imitators remain inactive, since the profit flow to be earned following entry becomes relatively smaller.

Lemma 3 *In the subgame with* $k \in \{3, ..., n-1\}$ *outsiders, if* $c_p \geq c_{k-1}^*$*, the* k *outsiders mix between actions* p *and* w*. Counting from the date at which the subgame is reached, the time of first entry converges as* Δ *goes to zero to an exponential distribution with parameter* $k\lambda_k$*, where* $\lambda_k \equiv r(\Pi_n - c_i)/((k-1)c_i)$*. Outsiders expect to gain* $O_k = \Pi_n - c_i$*, whereas insiders expect* $I_k = \mu_k \Pi_{n-k} + (1-\mu_k)\Pi_n$*, where* $\mu_k \equiv r/(r + k\lambda_k)$*.*

We now consider the more complex case with k outsiders and $c_p < c_{k-1}^*$. It is essential for our purposes to define J, the critical number of outsiders such that a war of attrition is played if the number of outsiders is *strictly less* than J (to be precise, for $k = 0, 1, ..., J-1$). Formally, we have $J = \inf\{k : c_p < c_{k-1}^*\}$. We will now show that for $J, ..., k, ..., n-1$

[11] Note that $\mu_k = (k-1)c_i/(k\Pi_n - c_i)$ is increasing in k, since $c_i < \Pi_n$ implies that $d\mu_k/dk = c_i(\Pi_n - c_i)/(k\Pi_n - c_i)^2 > 0$. Taking into account that both μ_k and $\Pi_{n-k} - \Pi_n$ are positive, the fact that Π_{n-k} and μ_k are both increasing in k then yields that $c_2^* < c_3^* < ... < c_{n-1}^*$.

($k \geq J$), a series of preemption games takes place. A priori, the players mix between the three available actions, w, p and u.[12] We denote $\rho_{a,k} \geq 0$ for the probability with which each outsider plays action a when k outsiders are left.

Recall that we are interested in equilibria where players can react instantaneously to each others actions, i.e in situations where the time Δ between successive play is small.[13] In what follows, we will not be deriving the exact play in a symmetric equilibrium for small values of Δ but consider an approximation of equilibrium play that is arbitrarily close to the true outcome.

More specifically, the approach will be the following. For a given length Δ for a period, in equilibrium, the mixing probabilities $\rho_{a,k}(\Delta)$ for $a \in \{w, p, u\}$ must be such that outsiders are indifferent between all three strategies (i.e., $V_{p,k} = V_{u,k} = V_{w,k}$) and such that these are indeed probabilities (i.e., $\rho_{a,k} \in (0,1)$ and $\rho_{u,k} + \rho_{p,k} + \rho_{w,k} = 1$). What we will do is to solve for the solution of this system for $\Delta = 0$, what we call the approximation of the equilibrium outcome,[14] and we will show that this solution exists and is unique. Given that the value functions are continuous in Δ and in the probabilities, this will be a close approximation of the equilibrium outcome for small enough values of Δ.

To illustrate further this method, consider the case of three players solved in Lemma 2. In that case we solved explicitly, for a fixed value of Δ, for the probabilities $\rho_{a,3}(\Delta)$, $a \in \{w, p, u\}$. In that case, we see from the solution presented in the proof of Lemma 2, that taking the limit of all the probabilities as Δ converges to zero (as we did) leads to the same solution as directly solving the system consisting of equations (1)-(3) for $\Delta = 0$, as was to be expected due to the continuity of the system.

We formally show in the proof of Lemma 4 below that the symmetric MPE can be approximated for small enough values of Δ by an equilibrium where the action of entering without protection is played with zero probability, i.e., $\rho_{u,k} = 0$. Thus, in the approximation we consider, the players will only mix between actions w and p. We denote $\rho_k = \rho_{p,k}$ for the individual probability of entry (so we have $\rho_{w,k} = 1 - \rho_k$). Letting $C^l_{k-1} = \binom{k-1}{l}$ denote the binomial coefficient indexed by $k-1$ and l, we can calculate the value of choosing action p as follows:

$$V_{p,k} = \sum_{l=0}^{k-1} C^l_{k-1} (\rho_k)^l (1 - \rho_k)^{k-1-l} I_{k-1-l}.$$

[12] We will show when proving Lemma 4 that, in the limit as Δ goes to zero, action u is not chosen.

[13] As emphasized by Fudenberg and Tirole (1991) when dealing with preemption games, a continuous-time version of the game cannot be directly used, and one is forced either to use approximations based on discrete-time games or to properly expand strategy spaces to accommodate for such approximations, as done by Fudenberg and Tirole (1985).

[14] Formally, what we mean by approximation of the equilibrium is a set of admissible mixing probabilities $\rho_{a,k}$ ($a \in \{p, u, w\}$) satisfying the following property: for any $\epsilon > 0$, there exists Δ_ϵ such that $\Delta < \Delta_\epsilon$ implies that $|\rho_{a.k}(\Delta) - \rho_{a,k}| < \epsilon$, where $\rho_{a.k}(\Delta)$ is the exact equilibrium play.

The value of paying the protection cost when entering depends on how many other outsiders simultaneously enter. If l other outsiders enter, the firm participates in the next period as an insider in a subgame with $k-1-l$ outsiders. His expected gain in this case is thus I_{k-1-l}. Overall, $V_{p,k}$ is the expected value to action p given the distribution of entry decisions of other outsiders.

Each of the k outsiders will mix between p and w so as to leave others indifferent between these two actions, which yields

$$V_{p,k} - c_p - c_i = \Pi_n - c_i,$$

since $V_{w,k} = \Pi_n - c_i$ for $\Delta = 0$. Letting $\overline{I}_{k-1-l} = I_{k-1-l} - \Pi_n$ and

$$F_k(\rho) = \sum_{l=0}^{k-1} C_{k-1}^{l} \, \rho^l (1-\rho)^{k-1-l} \, \overline{I}_{k-1-l},$$

the indifference condition can be equivalently written as:

$$F_k(\rho_k) = c_p.$$

Thus, we have in subgames with k imitators left to enter (and such that $c_p < c^*_{k-1}$) that the approximate mixing probability (provided it exists) must solve $F_k(\rho_k) = c_p$. Largely inspired by Vettas (2000), we now exploit the recursive nature of the problem and the properties of $F_k(\cdot)$. We show that the symmetric MPE of the game can be approximated for small values of Δ by an equilibrium such that outsiders mix between actions p and w with strictly positive probabilities. Furthermore, in this approximation, the probability of playing action p in equilibrium decreases as the number of outsiders decreases.

The main properties of the $F_k(\cdot)$ functions, for $k \in \{J, ..., n-1\}$, are presented in the graph below.

It holds that $F_J(\rho)$ is strictly decreasing in ρ, with $F_J(0) > c_p > F_J(1)$. There is thus clearly a unique solution to $F_J(\rho) = c_p$, ρ_J. This is intuitive: Following entry by at least one outsider, preemptive motives disappear and a war of attrition is played thereafter (by definition of J). The speed of such a war of attrition is determined by the number of other outsiders who enter. Given our previous finding that the continuation payoff of an insider is lower in a war of attrition played by fewer outsiders, the best scenario is if no one else enters ($\rho = 0$), whereas the worst scenario is if everyone else enters ($\rho = 1$). The mixing performed in equilibrium is somewhere in between.

For $k > J$, the pattern is slightly different. In these cases $F_k(\rho)$ is no longer decreasing in ρ. It can be shown (see proof of Lemma 4) that the continuation payoff of an insider (net of Π_n) has an inverted-U shape as a function of k: $\overline{I}_{k-1} < \overline{I}_{k-2} < ... < \overline{I}_{J-1}$ and $\overline{I}_{J-1} > \overline{I}_{J-2} ... > \overline{I}_0$. So $F_k(\rho)$ also has an inverted-U shape as a function of ρ.

Furthermore, we can show that $F_J(0) > c_p > F_J(1)$.

There is additional structure that can be exploited. In particular, $F_{k+1}(\cdot)$ starts off below $F_k(\cdot)$, reaches its maximum when crossing $F_k(\cdot)$ and is then above $F_k(\cdot)$. A direct consequence is that the equilibrium ρ_k is increasing in k, an intuitive property. In these preemption games, players want to rush to enter to become one of the insiders during a war of attrition that will likely follow. There is however a risk of excessive entry ex post. In a subgame where many players have already entered, and hence k is close to J, this risk becomes particularly severe, and the players in equilibrium therefore chose to enter with a lower probability. The following lemma formalizes all these ideas.

Lemma 4 *In subgames with $k \in \{J, ..., n-1\}$ outsiders, if $c_p < c^*_{k-1}$, then, for small enough Δ, the symmetric MPE can be approximated by the following equilibrium:*

(i) Outsiders mix only between actions p and w, and the probability ρ_k of playing p is uniquely defined by $F_k(\rho_k) = c_p$.

(ii) ρ_k is increasing in k.

(iii) Quasi-instantaneous entry by at least one outsider occurs.

We have therefore fully characterized the dynamics of imitation and protection in the case where $\Pi_n < c_i$. In the next sections we draw a number of important implications of these results.

4 Profits outside patents

We can now characterize profits of innovators in the absence of legal protection or in cases where they strategically choose not to patent. We show that the profits depend on the type of protection technologies available and can be very high even for expensive and bad technologies.

Proposition 1 *The equilibrium payoff of the innovator is bounded between Π_n and $\Pi_1 - \Pi_2$. Furthermore, there exist protection technologies (c_i, c_p) such that the innovator's payoff can be arbitrarily close to these bounds.*

Depending on the characteristics of the protection technology (c_i and c_p), the payoff to the innovator can be as low as Π_n or as high as $\Pi_1 - \Pi_2$. In the first case, the rents of the innovator are completely dissipated in the absence of patents. In the second case, even though we considered an environment a priori unfavorable to innovators, where imitation is instantaneous and protection technologies do not increase drastically the imitation cost ($c_i < \Pi_n$), we see that the payoff to the innovator can be very high in the absence of legal protection.

To understand more specifically the result, it is important to examine how the profits of the innovator vary with the characteristics of the protection technology. Below, we plot, for a given value of c_i, how the innovator's payoff (net of the protection cost) varies with c_p. The value of c_i determines the increasing sequence $\{c_k^*\}_{k=2}^{n-1}$. From the results of the previous section, we know that if c_p is in the interval (c_{k-1}^*, c_k^*), then in equilibrium, $n-1-k$ outsiders quasi-instantaneously enter, while the k remaining outsiders then play a war of attrition. The payoff to the innovator is then capped by the value I_k, the value of an incumbent when k outsiders remain (as illustrated in the graph). This value can be close to Π_n if c_p is close to the upper limit of the interval or can be quite high if it is close to the lower limit.

Interestingly, the maximum profit $\Pi_1 - \Pi_2$ is attained for a protection technology that does not perform very well (c_i close but less than Π_n) and is expensive $c_p > \Pi_2$. The idea is the following: when c_i is close to Π_n the war of attrition is such that the expected first entry is very late, since the payoff to the loser of the war is close to zero while the winners earn a positive payoff. Imitators thus initially have high incentives to enter immediately, preempt the others and benefit from this delay. However, the high cost of the protection technology ($c_p > \Pi_2$) renders this strategy unprofitable, even for one imitator, and hence all imitators are forced to play the war of attrition. The innovator's payoff converges to Π_1 as c_i approaches Π_n, but she has to pay a protection cost that, in the most favorable case, is just above Π_2. For such a technology, the payoff to the innovator is thus high even though there is no legal protection. The innovator also makes a significantly higher payoff than imitators, so innovation can lead to a competitive advantage despite imitation. The point is that the innovator can benefit from the imitators' incentive to free ride on each other.[15]

Our model can thus explain why innovation flourished in certain sectors where patent protection was not available. Consider the software industry. Until recently, software was not covered by patents and it was common for inventors to obfuscate the code: in other words, transforming the readable source code into code difficult to use directly. Today, various techniques are available and appear to be relatively cheap (low c_p). But this was not always the case and our theory could help explain why, even though patents did not apply, this was nevertheless an industry characterized by relentless innovation (see Boldrin and Levine (2007)).

Our results can also help us understand why in certain sectors, patents are much less popular than secrecy. According to surveys of managers (Cohen et al. 2000), electronic components and semiconductors are amongst the sectors where patents are judged to be least effective (the software industry is not part of the survey). Of course there is no data

[15]It is interesting to note that if the innovator could choose her protection technology, and this was then the only technology available to imitators, she would never choose an intermediate technologies, that raise the imitation cost to values in the range (Π_n, Π_2). Indeed, with a technology such that $c_i = \Pi_k$, $k \in (2, .., n)$, the revenues are at best Π_k, which is less than $\Pi_1 - \Pi_2$. The innovator then prefers to choose a technology that allows cheaper imitation and creates for the imitators a strategic motive to delay entry.

measuring c_i or c_p. Interestingly though, these two sectors are those where protection technologies can be most commonly observed. For instance, hardware obfuscation is a technique by which the description or the structure of electronic hardware is modified to intentionally conceal its functionality, making it significantly more difficult to reverse engineer.[16] We also observe technologies for sale that allow protection of integrated circuits from reverse engineering.[17] Our results provide a potential explanation for the variations across sectors of the propensity to patent. We hope our theoretical work can be used as a framework to conduct further empirical analysis of this question.

5 Design of patent policy

In the previous sections we have interpreted the investment in protection as paying to add technological components making the product difficult to copy. However applying for a patent can also be seen as an investment in protection. The cost c_p is then the cost of applying for the patent, while c_i is the cost for imitators of inventing around the patent.[18] Note that c_i can be seen as a measure of the breadth of the patent (see Denicolò (1996) for a definition of this concept). Empirical analysis suggest that in the electronics industries, patents increase imitation costs by only 7 percent (Mansfield et al (1981)), suggesting that c_i could be quite small.

The literature on the design of patent protection has generally focused on the length and breadth of the patent as tools to provide rents for the innovator (see e.g. Gilbert and Shapiro (1990), Klemperer (1990), Gallini (1992) or Denicolò (1996)). The cost of the patent—or more specifically the level of renewal fees—, is only considered in the literature as a tool, in combination with the length, for an uninformed designer to screen among innovations of different value. One exception is the paper by Hunt (2006), that shows that higher patent costs can increase investments in $R\&D$ when the exogenously given degree of overlap of the innovating firms is sufficiently important. Our paper, based on a completely different mechanism, shows that, surprisingly, increasing the cost of patenting can be a way to yield profits for innovators.

It can of course be argued that c_i, the cost of inventing around the patent will increase each time an imitator enters. We plan to examine a variant of the model including this feature.

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This interpretation of our model thus provides potentially testable implications on the effect of increasing the patenting cost c_p. In particular we find that an in c_p can increase

[16] There are other techniques, some cryptography-based.

[17] An example is given by the United States patent 7128271, described as "a semiconductor integrated circuit having a reverse engineering protection part that can be easily implemented".

[18] Note that according to Cohen et al., ease of inventing around is most often cited by managers as the reason for not patenting.

the profits an innovator can expect from applying for a patent and thus make her more likely to choose patenting over secrecy. However, the overall effect of an increase in c_p on the total number of patents granted is ambiguous as, even though more innovators would patent, later "imitators" would apply for fewer patents.

There is a growing body of empirical work (surveyed in de Rassenfose and van Pottelsberghe de la Potterie (2010)) examining the influence of patenting fees on patenting behavior. Most of these studies show that an increase in fees tends to decrease the number of patents (see for instance Eatum and Kortum (1996), Nicholas 2011). However, to test more directly our prediction, we need empirical evidence of patenting strategies of the "original innovators". Moser (2011) shows, using data on 19th century world fairs where exhibitors were presumably only genuine inventors, that while fees for carrying the patent to full term were \$37000 in Britain compared to \$612 in the US, the respective patented innovation represented 11.1 percent for Britain and 15.3 in the US, suggesting a small effect of patenting costs. Note also that Nicholas (2011), even though he demonstrates that a decrease in fees increased the number of patents granted, also shows that it decreased the patent quality, which is consistent with the mechanism we present.

We attempt to examine further the validity of our prediction by studying the correlation between the level of patenting fees and the percentage of innovators reporting to have used patents to protect their innovations in the European innovation survey CIS.[19] Using a 4 percent interest rate to calculate the present value of fees of a patent held to full term, we find a positive and significant correlation of 0.46 between patenting fees and use of patents by innovators (see Table 1). Of course more systematic empirical work is needed, but this preliminary evidence confirms our potentially surprising effect of fees on innovator's profits and furthermore underlines the fact that empirical work should attempt to distinguish patents by original inventors from patents by imitators who invent around the original patent.

6 Worker mobility

In the previous sections we focused on cases where the innovator and the initial imitators could protect themselves form further imitation by either making their technology hard to reverse engineer or by applying for a patent. However, knowledge of how to reproduce the technology is embodied in the researchers who developed it. Another key dimension of protection is therefore paying wages sufficiently high so that these researchers do not leave the firm. In the current section we examine the dynamics of wages. We show that this is a way of making the protection cost c_p endogenous.

[19] We use the aggregate data from CIS 4, reported in Table 2 of the Eurostat document "Innovative enterprises and the use of patents and other intellectual property rights". For patent fees, we use the data reported in the webpage of the European Patent Office.

There is a growing literature examining the mobility of scientists and the associated diffusion of knowledge. Lewis and Yao (2006), in a situation where some ideas developed in one firm can be potentially more useful in another, show how allowing ex ante for mobility of researchers can be optimal from the innovating firm's point of view. Kim and Marschke (2005) examine a model where innovators can choose between patenting and secrecy and show, both theoretically and empirically, that patents become more attractive when there is a high risk of scientists leaving with firms secrets. Franco and Mitchell (2008) compare situations where clauses restraining worker mobility can be included in contracts to cases where this is illegal.[20] The novelty of our approach is that it allows us to study fully dynamic aspects of employment in situations where firms can imitate in two distinct ways: in-house research or poaching a scientist from a different firm.[21]

We modify our model in the following way to address the question. We suppose that there are $n+1$ firms. Firm i is considered to be a pair composed of a financier f_i and researcher r_i. As in our previous model, the game starts with a firm, that we denote firm 1, having innovated. Researcher r_1 therefore has the knowledge of how to reproduce the invention. A firm that has not yet imitated and decides to do so can do it in two ways. It can hire a researcher of a previously successful firm or use its in house researcher, who is uninformed and can develop the invention at cost c_i.

We consider a specific model for the hiring process. After a firm has successfully developed an invention or successfully imitated, a second price auction for the researcher is run between this firm and all the remaining outsiders. The winning bidder hires the worker and pays the second highest bid. We suppose furthermore that if the winning bidder is the current employer of the researcher, then this researcher will stay with the firm forever. A natural interpretation of this assumption is that the current firm is participating in the bidding to make the researcher sign a non competition clause (covenant to compete). Thus, in that case, the firms not having yet imitated can only do so through in house research at cost c_i.

We present the results in the case where $c_i < c_2^*$ that illustrates the main intuitions.[22]

Proposition 2 *If $c_i < c_2^*$, then if the number of remaining imitators:*

1. *is larger than 4, one of the imitators wins the auction and pays the researcher $c_2^* + c_i$*
2. *is equal to 3, one of the imitators wins the auction and pays the researcher between c_2^* and $c_2^* + c_i$*

[20] They show that these so called "covenant to compete" can explain the initial advantage of Massachusetts' Route 128 and the subsequent overtaking by the Sillicon Valley.

[21] Note that contrary to most papers in the literature, a notable exception being Franco and Mitchell (2008), we focus on a case where the creation of a spin-out by a researcher leaving the firm decreases total profits, i.e does not create a differentiated product, although this case could be considered in our framework.

[22] The general case presents similar dynamics.

3. *is equal to 2, the researcher stays with the current employer for a bonus between 0 and c_2^* and the remaining two imitating firms play a war of attrition*

4. *is equal to 1, the researcher moves for a zero wage*

Furthermore, *the profits of the innovator are:* $I_2 = \mu_2 \Pi_{n-2} + (1 - \mu_2)\Pi_n$, *where* $\mu_2 \equiv r/(r+2\lambda_2)$ *and the original researcher accumulates salaries of at least* $c_2^* + (n-3)(c_2^* + c_i)$.

We first observe that the timing of imitation is the following: $n-2$ firms successively and quickly enter by winning the auction and hiring the informed researcher. At this point, when only 2 imitators are left, the last firm having entered will pay a sufficiently high bonus so as to win the auction and keep the researcher. The remaining two imitators can thus only enter by conducting in house research at cost c_i and will wait to do so in the hope that their competitor does it before them and that they can subsequently hire the informed researcher at a zero wage.

We note that this path of entry is very similar to the one identified in the general model of section 3 for the case $c_p < c_2^*$. In that case a series of preemption games were played and when two imitators were left, they played a war of attrition with the same probability of entry λ_2. The main difference is that in the current setting we do not have the preemption phase. Outsiders still have an incentive to enter quickly, but the auction solves the mis-coordination problem characterizing preemption games, since the auction determines a unique winner.

Proposition 3 can be seen as providing a microeconomic foundation for our assumption of a fixed protection cost. Indeed, as long as the number of outsiders is greater than four, the imitators who enter pay a premium of c_2^* above the imitation cost c_i, premium that can be interpreted as the cost of protection. The intuition is the following: the early imitators, by paying the premium c_2^*, are purchasing the right to benefit from the delay in the war of attrition. The order of entry among those does not influence their expected profits since with the auction there is no mis-coordination. All the initial entrants therefore pay the same price for entry $c_i + c_2^*$.

We characterize the level of bonuses on the equilibrium path. Naturally, as the number of remaining imitators decreases, the bonus that the researcher obtains decreases. We note that neither the innovator, nor the imitators, attempt to keep their researcher until only two imitators are left. The intuition is the following: the innovator and the initial imitators are free riding on the protection effort of the $n-3$rd entrant. In previous subgames, there is no point in trying to keep the researcher since the remaining imitators will in any case try to rush to enter, and if they cannot do it by hiring a researcher, they will do it through in house research. Keeping the researcher has even a negative effect on expected profits since it opens the way to a preemption game with a risk of mis-coordination and excessive entry.

7 Conclusion

In this paper we show that introducing in canonical models of innovation the possibility that inventors can invest to make their products difficult to reverse engineer challenges the received wisdom on the need for intellectual property protection. Surprisingly the protection technologies that yield the highest returns for the innovator are expensive and do not protect very well. We also show our model has implications for the design of the patent system and for the patterns of employment in innovative industries.

We believe our model and results could be the basis for interesting empirical work. At the very least it underlines the need for more comprehensive data on two dimensions. First, little is still known on the cost of reverse engineering inventions, and how these costs vary by industry. Second, little information is available on protection technologies, their cost, the level of protection they confer. Although there is a large body of anecdotal evidence showing that technological protection is commonly used, there is no systematic measurement allowing for more detailed empirical analysis.

Finally, we want to suggest that our model can also contribute to the understanding of the path of diffusion of innovations. Starting with the seminal paper by Griliches (1957), numerous papers have documented the fact that the pattern of adoption of new technologies is typically S-shaped: slow initial adoption is followed by a quick acceleration and then slowing down.[23] If we view the process of imitation as a process of adoption of a technology, our paper provides a different theoretical foundation for the delay in adoption.[24] Firms wait to adopt in the hope that the technology will enter the public domain at some point. Of course the path is more a step function than a smooth S-shape. We could however imagine introducing uncertainties, for instance in the time needed to obtain an invention after having paid the imitation cost, that could generate a smoother path. This could be the object of interesting future work.

[23]There are numerous papers proposing a theoretical explanation for this pattern of adoption. Some are non strategic and often based on models of diffusion of information. Others consider firms that are strategic in their adoption decisions (Reinganum, Fudenberg and Tirole (1985), Ruiz Aliseda and Zemsky (2006)).

[24]Note that the empirical literature is not explicit about what is the process of adoption of a technology, whether it is purchasing from the inventor or whether it comes through imitation.

Appendix

Proof of Lemma 1. Fudenberg and Tirole (1991) show that the unique symmetric equilibrium of the discrete-time war of attrition with short period lengths converges to the unique symmetric equilibrium of the war of attrition in continuous time. This leads us to prove the result using the continuous-time version of the game.

Consider the expected payoff of a firm if she chooses to enter at time τ_2 given that the other imitator chooses their entry time according to an atomless and gapless distribution $F_2(\cdot)$ with full support on $[0, \infty)$ and density $f_2(\cdot)$. Given that the other firm has made an unknown draw from $F_2(\cdot)$, a firm that enters at τ_2 expects to gain

$$\widehat{V}_2(\tau_2) = \int_0^{\tau_2} \Pi_n e^{-rs} dF_2(s) + \int_{\tau_2}^{\infty} (\Pi_n - c_i) e^{-r\tau_2} dF_2(s).$$

In a mixed-strategy Nash equilibrium, the firm should be indifferent among all possible entry times, which formally means that we should have that $d\widehat{V}_2(\tau_2)/d\tau_2 = 0$ for all $\tau_2 \geq 0$. Straightforward differentiation using that $\int_{\tau_2}^{\infty} dF_2(s) = 1 - F_2(\tau_2)$ yields that

$$\frac{d\widehat{V}_2(\tau_2)}{d\tau_2} = e^{-r\tau_2}[c_i f_2(\tau_2) - r(\Pi_n - c_i)(1 - F_2(\tau_2))].$$

Letting $h_2(\tau_2) \equiv f_2(\tau_2)/(1-F_2(\tau_2))$ denote the hazard rate of $F_2(\cdot)$ and equating $d\widehat{V}_2(\tau_2)/d\tau_2$ to zero yields that the hazard rate is constant and equal to $h_2(\tau_2) = r(\Pi_n - c_i)/c_i$, so $F_2(\tau_2) = 1 - e^{-\lambda_2 \tau_2}$, where $\lambda_2 \equiv r(\Pi_n - c_i)/c_i$. Given that a probability distribution is exponential if and only if its hazard rate is constant, the individual entry time follows an exponential distribution with parameter $\lambda_2 = r(\Pi_n - c_i)/c_i$.

Furthermore, since a firm is indifferent among all the pure strategies played with positive density, the expected gain of an outsider converges to $O_2 = \Pi_n - c_i$.

We have shown that both outsiders make independent draws from an exponential distribution with the same hazard rate λ_2, so the time $\widehat{\tau}$ of first entry must be exponentially distributed with parameter $2\lambda_2$. The expected payoff for an insider is therefore given by:

$$I_2 = \int_0^{\infty} \left(\int_0^{\widehat{\tau}} \pi_{n-2} e^{-rs} ds + \int_{\widehat{\tau}}^{\infty} \pi_n e^{-rs} ds \right) 2\lambda_2 e^{-2\lambda_2 \widehat{\tau}} d\widehat{\tau}.$$

Integrating and letting $\mu_2 \equiv r/(r + 2\lambda_2)$ yields

$$I_2 = \mu_2 \Pi_{n-2} + (1 - \mu_2)\Pi_n.$$

■

Proof of Lemma 2. (i) As indicated in the main text action p is weakly dominated if $c_p \geq c_2^*$. The insiders mix every period between u and w. Suppose that two firms draw their time of imitation with an unprotected technology using an atomless and gapless distribution function $F_3(\cdot)$ with full support on $[0, \infty)$. Denoting these (random) draws by s and s', we have that the expected payoff of a firm if he imitates at time τ_3 with probability one (conditional upon no other firm imitating earlier) is

$$\begin{aligned}\widehat{V}_3(\tau_3) &= \int_0^{\tau_3} \Pi_n e^{-rs} f_3(s)(1 - F_3(s))ds + \int_0^{\tau_3} \Pi_n e^{-rs} f_3(s')(1 - F_3(s'))ds' \\ &\quad + (1 - F_3(\tau_3))^2(\Pi_n - c_i)e^{-r\tau_3}\end{aligned}$$

Because it must hold that $d\widehat{V}_3(\tau_3)/d\tau_3 = 0$ for all $\tau_3 \geq 0$, straightforward computations show that we must have $h_3(\tau_3) \equiv f_3(\tau_3)/(1 - F_3(\tau_3)) = r(\Pi_n - c_i)/(2c_i)$. Hence, $F_3(\tau_3) = 1 - e^{-\lambda_3\tau_3}$, where $\lambda_3 \equiv r(\Pi_n - c_i)/(2c_i)$. Each of the imitators left expects to gain $O_3 \equiv \Pi_n - c_i$ (since $\widehat{V}_3(\tau_3) = \Pi_n - c_i$ for $\tau_3 = 0$). In turn, the fact that the time at which imitation without protective measures takes place is exponentially distributed with parameter $3\lambda_3$ yields that the payoff expected by the firms active in the market with a protected technology is

$$I_3 = \mu_3\Pi_{n-3} + (1 - \mu_3)\Pi_n,$$

where $\mu_3 \equiv r/(r + 3\lambda_3)$.

(ii) We now consider the case $c_p < c_2^*$. In principle, firms will mix using the three actions available to each of them, namely w, p and u. We denote $\rho_{a,k} \geq 0$ for the probability with which one of the outsiders plays action a when k outsiders remain to enter. We let $V_{a,k}$ denote the outsider's payoff when following action $a \in \{w, p, u\}$. We must have that $V_{w,3} = V_{p,3} = V_{u,3}$ in a mixed-strategy equilibrium, where

$$V_{p,3} = \rho_{w,3}^2(\pi_{n-2}\Delta + I_2\delta^\Delta) + 2\rho_{w,3}(1 - \rho_{w,3})(\pi_{n-1}\Delta + \Pi_n\delta^\Delta) + (1 - \rho_{w,3})^2\Pi_n - (c_i + c_p) \quad (1)$$

$$V_{u,3} = \rho_{w,3}^2(\pi_{n-2}\Delta + \Pi_n\delta^\Delta) + 2\rho_{w,3}(1 - \rho_{w,3})(\pi_{n-1}\Delta + \Pi_n\delta^\Delta) + (1 - \rho_{w,3})^2\Pi_n - c_i \quad (2)$$

and

$$V_{w,3} = \rho_{w,3}^2(V_{w,3}\delta^\Delta) + 2\rho_{w,3}\rho_{p,3}O_2\delta^\Delta + (\rho_{w,3} + \rho_{p,3} + 1)\rho_{u,3}\Pi_n\delta^\Delta + \rho_{p,3}^2(\Pi_n - c_i)\delta^\Delta \quad (3)$$

Because $V_{p,3} = V_{u,3}$, it holds after using the fact $\rho_{w,3} \geq 0$ that

$$\rho_{w,3} = \sqrt{\frac{c_p}{(I_2 - \Pi_n)\delta^\Delta}}$$

Using the working hypothesis that $c_p < c_2^* \equiv I_2 - \Pi_n$ yields that $\dfrac{c_p}{(I_2^* - \Pi_n)\delta^\Delta} < \delta^{-\Delta}$, so $\rho_w \leq 1$ for $\Delta > 0$ close enough to zero.

Because $\rho_{u,3} = 1 - (\rho_{w,3} + \rho_{p,3})$ and $O_2 = \Pi_n - c_i$, the expression for $V_{w,3}$ can be rewritten as follows:

$$V_{w,3} = \frac{(1-\rho_{w,3}^2)\Pi_n - \rho_{p,3}(\rho_{p,3} + 2\rho_{w,3})c_i}{\delta^{-\Delta} - \rho_{w,3}^2}.$$

Equating $V_{u,3}$ and $V_{w,3}$ yields the value for $\rho_{p,3} \geq 0$. We find, for small $\Delta > 0$ that

$$\rho_{w,3} \approx \sqrt{\frac{c_p}{\mu_2(\Pi_{n-2} - \Pi_n)}},$$

$$\rho_{p,3} \approx 1 - \sqrt{\frac{c_p}{\mu_2(\Pi_{n-2} - \Pi_n)}},$$

and

$$\rho_{u,3} \approx 0.$$

To make exposition notationally simpler, let us normalize to zero the date at which the subgame with three imitators starts. Given m periods of play between time 0 and some fixed time $t > 0$, it holds that the probability that no firm has imitated and protected her technology once time t has elapsed is $(\rho_{w,3})^{3m} = (\rho_{w,3})^{3t/\Delta}$ (since $m = t/\Delta$), which converges to zero as Δ converges to zero for any arbitrarily chosen $t > 0$. We then must have that there is probability one that at least one imitator will imitate and protect his technology (almost) instantaneously.

We conclude the proof by characterizing the probability distribution over entry outcomes at time 0 and equilibrium payoffs. Because the probability of no entry at any point in time is $(1-\rho_{p,3})^3$, it holds that the probability that at least one imitator enters is $1-(1-\rho_{p,3})^3$. Conditional upon at least one firm entering, we then have that $\phi_3(3) = (\rho_{p,3})^3/(1-(1-\rho_{p,3})^3)$, $\phi_3(2) = 3(1-\rho_{p,3})(\rho_{p,3})^2/(1-(1-\rho_{p,3})^3)$ and $\phi_3(1) = 3(1-\rho_{p,3})^2\rho_{p,3}/(1-(1-\rho_{p,3})^3)$, where $\phi_k(l)$ denotes the probability that $l \geq 1$ imitators enter simultaneously at 0 given that there are $k \geq l$ of them. We finally observe that an outsider's continuation payoff at the beginning of these subgames is approximately $O_3 = \Pi_n - c_i$ (since $V_{p,3} = V_{u,3} = V_{w,3} \approx \Pi_n - c_i$ for small enough $\Delta > 0$). Since $I_1 = I_0 = \Pi_n$, the expected payoff earned by an insider is

$$I_3 = \phi_3(1)I_2 + (1-\phi_3(1))\Pi_n.$$

■

Proof of Lemma 3. We prove the result by induction. Lemma 2 established the result for $k = 3$, so it only remains to prove that it holds for $k \geq 4$ whenever it is true for $k-1$. So suppose that the result holds for $k-1$, and consider the subgames with k outsiders when $c_p \geq c^*_{k-1}$.

Let us focus on a player's incentive to enter and pay the protection cost at a certain period. Since $c^*_j < c^*_{k-1}$ for all $j < k-1$, he knows when choosing action p that action p

being simultaneously chosen by $l \geq 0$ other outsiders will result in the remaining outsiders playing a war of attrition (by the induction hypothesis). Clearly, the highest payoff that can be achieved is the one attained when no other player enters simultaneously, i.e., $l = 0$. Thus, the highest payoff he can obtain by entering and paying the protection cost is $I_{k-1} - c_p = \mu_{k-1}\Pi_{n-k+1} + (1 - \mu_{k-1})\Pi_n - c_p$. Since $c_p \geq c^*_{k-1}$ implies $I_{k-1} - c_p < \Pi_n$, it then follows that no outsider must be willing to enter by paying the protection cost in subgames with k outsiders.

The k outsiders will therefore mix between waiting and entering without protection. Suppose that the competitors draw their time of imitation with an unprotected technology using an atomless and gapless distribution function $F_k(\cdot)$ with full support on $[0, \infty)$. We then have that the expected payoff of a firm if he imitates at time τ_k with probability one (conditional upon no other firm imitating earlier) is

$$\widehat{V}_k(\tau_k) = (k-1)\int_0^{\tau_k} \Pi_n e^{-rs} f_k(s)(1 - F_k(s))ds + (1 - F_k(\tau_k))^{k-1}(\Pi_n - c_i)e^{-r\tau_k}$$

In order for such a firm to be indifferent between all the possible imitation times, it is easy to show that we must have that $F_k(\tau_k) = 1 - e^{-\lambda_k \tau_k}$, where $\lambda_k \equiv r(\Pi_n - c_i)/((k-1)c_i)$. Each of the outsiders expects to gain $O_k \equiv \Pi_n - c_i$ (since $\widehat{V}_k(\tau_k) = \Pi_n - c_i$ for $\tau_k = 0$). In addition, because the time at which the first imitation takes place is exponentially distributed with parameter $k\lambda_k$, the expected profit of an insider is given by

$$I_k = \mu_k \Pi_{n-k} + (1 - \mu_k)\Pi_n,$$

where $\mu_k \equiv r/(r + k\lambda_k)$. ■

Proof of Lemma 4. We first show that we have $\rho_{u,k} = 0$. We show this result by induction starting at $k = J$. For $\Delta = 0$, we have

$$V_{u,J} = \Pi_n - c_i$$

and

$$\begin{aligned} V_{w,J} &= \Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0]\ V_{w,J} + \\ &\sum_{m=1}^{J-1}\sum_{l=0}^{J-1-m} \Pr[X_{w,J} = J-1-l-m, X_{p,J} = l, X_{u,J} = m]\ \Pi_n + \\ &\sum_{l=1}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0]\ O_{J-l}, \end{aligned}$$

where $\Pr[X_{w,k}, X_{p,k}, X_{u,k}]$ denotes the probability that $X_{w,k}$ outsiders happen to choose w, $X_{p,k}$ outsiders happen to choose p and $X_{u,k}$ outsiders happen to choose u. We know that for all $j < J$, a war of attrition is played and, according to lemma 3, $O_j = \Pi_n - c_i$, so the system of equations can be rewritten as

$$V_{u,J} = \Pi_n - c_i$$

and

$$\begin{aligned} V_{w,J} &= \Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0]\ V_{w,J} \\ &+ (1 - \Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0])\ \Pi_n \\ &- \sum_{l=1}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0]\ c_i \end{aligned}$$

In a mixed strategy equilibrium, the player is indifferent between all strategies in the support, so we must have $V_{u,J} = V_{w,J}$, which implies that

$$\sum_{l=1}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0]\ /\ (1 - \Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0]) = 1.$$

This holds if and only if

$$\sum_{l=0}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0] = 1,$$

whence we get that $\rho_{u,J} = 0$. Furthermore, this implies that $O_J = \Pi_n - c_i$, and the property is therefore true for $k = J$. The reasoning follows exactly the same lines for larger values of k.

After showing that $\rho_{w,k} + \rho_{p,k} = 1$, we proceed to pin down the value taken by the mixing probability $\rho_k \equiv \rho_{p,k}$. Consider first the "last preemption game", i.e the subgame where J outsiders are left to enter. As shown in the main text, the indifference between actions p and w is defined by

$$F_J(\rho_J) = c_p,$$

where

$$F_J(\rho) = \sum_{l=0}^{J-1} C_{J-1}^{l}\ \rho^l (1-\rho)^{J-1-l}\ \overline{I}_{J-1-l}.$$

Note that following entry by at least one outsider, a war of attrition is played (by definition of J). The speed is determined by the number of other insiders who enter. Note that according to Lemma 3, $\overline{I}_{J-1-l} = \mu_{J-1-l}[\Pi_{n-(J-1-l)} - \Pi_n] = c^*_{J-1-l}$. We showed previously that c^*_k is an increasing function of k. So we have $\overline{I}_{J-1} > \overline{I}_{J-2} > ... > \overline{I}_0$. So it can be immediately observed that $F_J(\rho)$ is a strictly decreasing function of ρ. Indeed increasing ρ shifts the distribution to states where the payoff is lower.

Furthermore, $J = \inf\{k : c_p < c^*_{k-1}\}$ implies that $F_J(0) = \overline{I}_{J-1} = c^*_{J-1} > c_p$. Since

$F_J(1) = \overline{I}_0 = 0$ and $F_J(\rho)$ is a continuous and strictly decreasing function, it then follows that the equation $F_J(\rho) = c_p$ has a unique solution $\rho_J \in (0,1)$.

Just as we did in the proof of Lemma 2, it can be shown that

$$\phi_J(l) = C_J^l \frac{(\rho_J)^l (1-\rho_J)^{J-l}}{1-(1-\rho_J)^J},$$

which is the conditional probability that l players enter simultaneously conditional on at least one entering.

We now work recursively with $F_{k+1}(\rho)$ for $k \geq J$. To do so, we first explore some key properties of $F_{k+1}(\rho)$. In particular, we claim that the following properties hold for $k \geq J$:

- Property 1:
$$\frac{\partial F_{k+1}}{\partial \rho}(\rho) = (\frac{k}{1-\rho})(F_k(\rho) - F_{k+1}(\rho)).$$
- Property 2:
$$\frac{\partial F_{k+1}}{\partial \rho}(0) > 0.$$
- Property 3:
$$\overline{I}_k = F_{k+1}(\rho_k).$$

Given these properties (which we show to hold below), the proof of the lemma will be complete. To see this, let $k \geq J$ throughout, and note from Properties 1 and 2 that we can conclude that $F_{k+1}(\rho)$ is increasing at zero, reaches a maximum when $F_{k+1}(\rho)$ and $F_k(\rho)$ cross and is then decreasing. Furthermore, we know that $F_{k+1}(1) = \overline{I}_0 = 0$. So to establish that $F_{k+1}(\rho) = c_p$ has a unique solution it is sufficient to show that $F_{k+1}(0) > c_p$. To prove it, note that we have $F_{k+1}(0) = \overline{I}_k$, and Property 3 implies that $\overline{I}_k = F_{k+1}(\rho_k)$, so it holds that $F_{k+1}(0) = F_{k+1}(\rho_k)$. Because $F_{k+1}(\rho)$ is increasing at zero according to Property 2, the unique maximum must be reached somewhere between 0 and ρ_k. According to Property 1, we know that $F_{k+1}(\rho) > F_k(\rho)$ for $\rho \geq \rho_k$, and therefore $F_{k+1}(\rho_k) > F_k(\rho_k)$. Taking into account that $F_{k+1}(\rho_k) = F_{k+1}(0)$, as we just showed, and that $F_k(\rho_k) = c_p$, it follows that $F_{k+1}(0) > c_p$, as desired.

So we can finish the proof of the lemma by just proving each of the properties described above. To show that Property 1 holds, note that we have

$$F_k(\rho) = \sum_{l=0}^{k-1} C_{k-1}^l \ (\rho)^l (1-\rho)^{k-1-l} \ \overline{I}_{k-1-l}$$

and

$$F_{k+1}(\rho) = \sum_{l=0}^{k} C_k^l \ (\rho)^l (1-\rho)^{k-l} \ \overline{I}_{k-l}.$$

So we can establish that

$$\begin{aligned} F_k(\rho) - F_{k+1}(\rho) &= \sum_{l=0}^{k-1} C_{k-1}^l \ (\rho)^l (1-\rho)^{k-1-l} \ \overline{I}_{k-1-l} - \sum_{l=0}^{k} C_k^l \ (\rho)^l (1-\rho)^{k-l} \ \overline{I}_{k-l} \\ &= \sum_{l=1}^{k} C_{k-1}^{l-1} \ (\rho)^{l-1} (1-\rho)^{k-l} \ \overline{I}_{k-l} - \sum_{l=1}^{k} C_k^l \ (\rho)^l (1-\rho)^{k-l} \ \overline{I}_{k-l} - (1-\rho)^k \ \overline{I}_k. \end{aligned}$$

Consider

$$\begin{aligned} \frac{\partial F_{k+1}}{\partial \rho}(\rho) &= \sum_{l=0}^{k} C_k^l \ \left[l(\rho)^{l-1}(1-\rho)^{k-l} - (k-l)(\rho)^l (1-\rho)^{k-l-1} \right] \ \overline{I}_{k-l} \\ &= \sum_{l=0}^{k} C_k^l \ (\rho)^{l-1} (1-\rho)^{k-l-1} \left[l - k\rho \right] \ \overline{I}_{k-l} \\ &= \sum_{l=1}^{k} C_k^l \ (\rho)^{l-1} (1-\rho)^{k-l-1} \left[l - k\rho \right] \ \overline{I}_{k-l} - k(1-\rho)^{k-1} \overline{I}_{k-l}. \end{aligned}$$

We develop the first term of this formula to obtain:

$$\frac{\partial F_{k+1}}{\partial \rho}(\rho) = \sum_{l=1}^{k} l C_k^l \ (\rho)^{l-1}(1-\rho)^{k-l-1} \ \overline{I}_{k-l} - k \sum_{l=1}^{k} C_k^l \ (\rho)^l (1-\rho)^{k-l-1} \ \overline{I}_{k-l} - k(1-\rho)^{k-1} \overline{I}_{k-l}.$$

Given that $C_{k-1}^{l-1} = l C_k^l / k$, and using the formula for $F_k(\rho) - F_{k+1}(\rho)$ established above, we have:

$$\frac{\partial F_{k+1}}{\partial \rho}(\rho) = \left(\frac{k}{1-\rho}\right)(F_k(\rho) - F_{k+1}(\rho)),$$

as claimed.

To prove Property 2, note that

$$\begin{aligned} \frac{\partial F_k}{\partial \rho}(\rho) &= \sum_{l=0}^{k-1} C_{k-1}^l \ \left[l(\rho)^{l-1}(1-\rho)^{k-1-l} - (k-1-l)(\rho)^l (1-\rho)^{k-l-2} \right] \ \overline{I}_{k-1-l} \\ &= \sum_{l=0}^{k-1} C_{k-1}^l \ (\rho)^{l-1} (1-\rho)^{k-l-2} \left[l - (k-1)\rho \right] \ \overline{I}_{k-1-l}, \end{aligned}$$

so

$$\frac{\partial F_k}{\partial \rho}(0) = k(1-(k-1))\overline{I}_{k-2} > 0.$$

In the last place, to prove Property 3, denote $\widehat{I}_k(\rho)$ for the expected payoff to an insider when there are k outsiders who choose to enter with probability ρ (the expectation being conditional upon at least one outsider entering). Then

$$\widehat{I}_k(\rho) = \sum_{l=1}^{k} C_k^l \, \frac{(\rho)^l (1-\rho)^{k-l}}{1-(1-\rho)^k} \, \overline{I}_{k-l},$$

so straightforward manipulations yield:

$$\begin{aligned}
(1-(1-\rho)^k) \, \widehat{I}_k(\rho) &= \sum_{l=1}^{k} C_k^l \, (\rho)^l (1-\rho)^{k-l} \, \overline{I}_{k-l} \\
&= \sum_{l=0}^{k} C_k^l \, (\rho)^l (1-\rho)^{k-l} \, \overline{I}_{k-l} - (1-\rho)^k \, \overline{I}_k \\
&= F_{k+1}(\rho) - (1-\rho)^k \, \overline{I}_k.
\end{aligned}$$

We know that $\widehat{I}_k(\rho_k) = \overline{I}_k$, so using the previous expression for $\rho = \rho_k$, we have

$$(1-(1-\rho)^k) \, \overline{I}_k = F_{k+1}(\rho_k) - (1-\rho)^k \, \overline{I}_k,$$

that is,

$$\overline{I}_k = F_{k+1}(\rho_k).$$

This completes the proof of Properties 1-3, and hence the proof of Lemma 4. ■

Proof of Prop 3.

We solve the model by backwards induction, using the notation O_k for the expected payoff of an outsider in a subgame with k outsiders left, I_k for the payoff of an insider who entered in one of the previous subgames and I_k^l for the payoff of the insider who just entered.

k = 1: Regardless of the result of bidding, the outsider will come in, so the insider bids his valuation, zero. Outsider gets the researcher for a zero wage. Payoffs are: $I_1 = I_1^l = \Pi_n$ and $O_1 = \Pi_n$

k = 2: We first study the subgame following an outcome of the auction such that the insider won the bidding. There are therefore two outsiders who can only enter by doing the research in-house by paying cost c_i. This leads to a war of attrition as in section 3. Payoffs are then $\mu_2 \Pi_{n-2} + (1-\mu_2)\Pi_n$ for the insider and $O_2 = \Pi_n - c_i$ for outsiders

We now examine the bidding strategies. The insider if he wins with bid w, gets $\mu_2 \Pi_{n-2} + (1-\mu_2)\Pi_n - w$ and if he loses Π_n. Weakly dominant strategy in a second price auction is for the insider to bid his valuation c_2^* (where $c_2^* = I_2 - \Pi_n = \mu_2[\Pi_{n-2} - \Pi_n]$) The payoffs for outsiders are the following: if he wins with bid w, gets $\Pi_n - w$, if he loses to insider $\Pi_n - c_i$ and if he looses to outsider Π_n. Since $c_2^* > c_i$, the insider by bidding his valuation c_2^*, wins and pays price between 0 and c_i, payoffs are $I_2 = \mu_2\Pi_{n-2} + (1-\mu_2)\Pi_n$, $I_2^l = \in [I_2 - c_i, I_2]$ and $O_2 = \Pi_n - c_i$

k $=$ **3**: We first study the subgame following an outcome of the auction such that the insider won the bidding. When an outsider comes in, he becomes the last insider in the next subgame and gets $I_2^l - c_i$, which is at least $I_2 - 2c_i$. If one of the competing outsiders enters, he gets $O_2 = \Pi_n - c_i$. Given that $c_2^* > c_i$ implies $I_2 - 2c_i > O_2 = \Pi_n - c_i$, this leads to a preemption motive and the expected payoff of the outsiders in such a subgame is $\Pi_n - c_i$ and the insiders obtain an expected payoff strictly less than I_2 (since there is a risk of miscoordination).

So if we consider the bidding, the insider knows that if he wins the bidding he gets a payoff strictly less than if he looses and gets I_2. He always bids zero.

The outsiders bid their value: $b = I_2^l - O_2 \in [c_2^*, c_2^* + c_i]$. We have for insiders $I_3 = I_3^l = I_2$, and for outsiders $O_3 = \frac{1}{3}(I_2^l - b) + \frac{2}{3}O_2 = \frac{1}{3}(I_2^l - (I_2^l - O_2)) + \frac{2}{3}O_2 = O_2$

k $=$ **4**: We first study the subgame following an outcome of the auction such that the insider won the bidding. When an outsider comes in, he becomes the last insider in the next subgame and gets $I_2^l - c_i$, which equals $I_2 - 2c_i$. If one of the competing outsiders enters, he gets $O_2 = \Pi_n - c_i$. Given that $c_2^* > c_i$, this leads to a preemption motive and the expected payoff of the outsiders in such a subgame is $\Pi_n - c_i$ and the insiders obtain an expected payoff strictly less than I_2 (since there is a risk of miscoordination).

So if we consider the bidding, the insider knows that if he wins the bidding he gets a payoff strictly less than if he looses and gets I_2. He always bids zero.

The outsiders bid their value: $b = I_3^l - O_3 = c_2^* + c_i$. We have for insiders $I_4 = I_4^l = I_2$, and for outsiders $O_4 = \frac{1}{4}(I_3^l - b) + \frac{3}{4}O_3 = O_3 = \Pi_n - c_i$

The result can therefore be shown by induction

■

Table 1

Country	Discounted patent cost	Proportion using patents
Belgium	2560	11
Danemark	4207	19.6
Germany	7488	20.1
Greece	4154	3
Spain	2854	11.8
France	3258	22.2
Italy	3870	13.4
Luxemburg	1717	8.8
Netherlands	6398	14.4
Portugal	3199	7
Finland	5265	18.2
Norway	7419	17.1

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